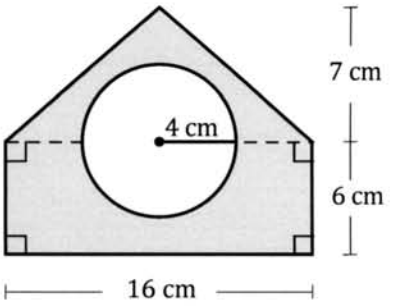
Chapter 8.

Perimeter and area.

Rectangles, parallelograms, triangles, trapeziums and circles.
The preliminary work section at the beginning of this book reminded you how to find the areas of rectangles, parallelograms, triangles, trapeziums and circles. With this knowledge you should also be able to determine the areas of shapes that are combinations of these shapes or that are parts of such shapes, as the following examples demonstrate.

Example 1
Find the area of the shaded shape shown on the right.



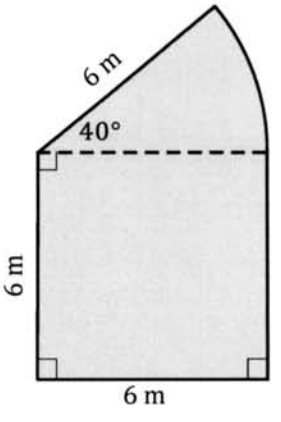
Area of rectangle $= 16 \text{ cm} \times 6 \text{ cm}$
$= 96 \text{ cm}^2$

Area of triangle $= \frac{16 \text{ cm}}{2} \times 7 \text{ cm}$
$= 56 \text{ cm}^2$

Area of circle $= \pi \times (4 \text{ cm})^2$
$= 16\pi \text{ cm}^2$

Thus, shaded area $= 96 \text{ cm}^2 + 56 \text{ cm}^2 - 16\pi \text{ cm}^2$
$= 101{\cdot}7 \text{ cm}^2$ (rounded to 1 d.p.)

Example 2
Find the area and the perimeter of the shaded shape shown on the right.



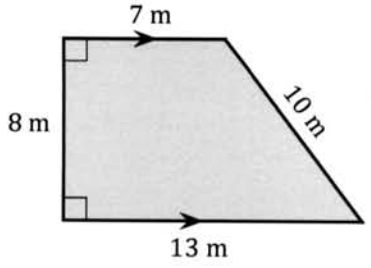
Perimeter of shaded shape $= 4 \times 6 \text{ m} + \frac{2 \times \pi \times 6 \text{ m}}{360} \times 40$
$= 28{\cdot}19 \text{ m}$ (to nearest cm).

Area of shaded shape $= 6 \text{ m} \times 6 \text{ m} + \frac{\pi \times (6 \text{ m})^2}{360} \times 40$
$= 48{\cdot}57 \text{ m}^2$ (rounded to 2 d.p.)

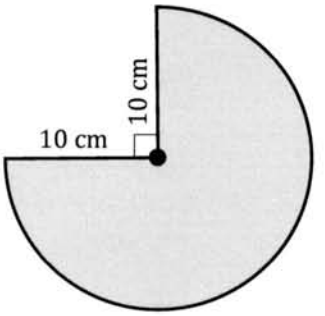
Exercise 8A.

For questions 1 to 8 find (a) the perimeter and (b) the area of the shaded region.

1.

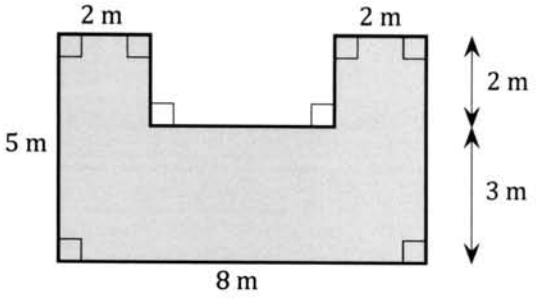


2.

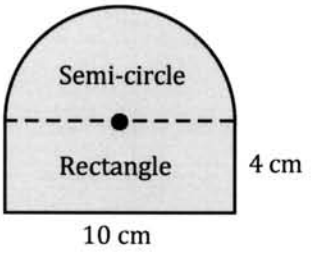


Give perimeter to the nearest mm and area in cm^2 to nearest cm^2.

3.

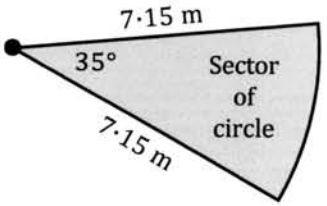


4.



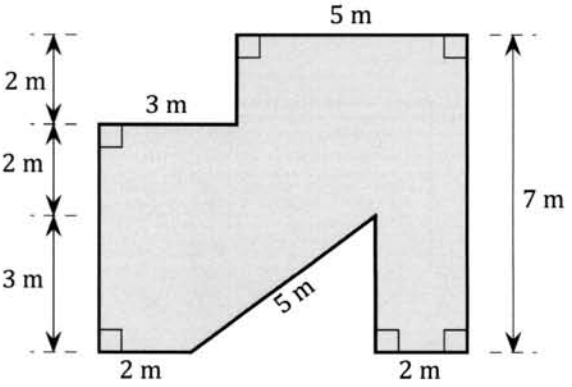
Give perimeter to the nearest mm and area in cm^2 to nearest cm^2.

5.

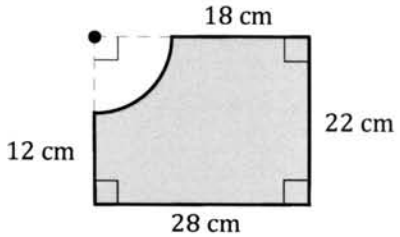


Give perimeter to the nearest cm and area in m^2 rounded to 2 d.p.

6.

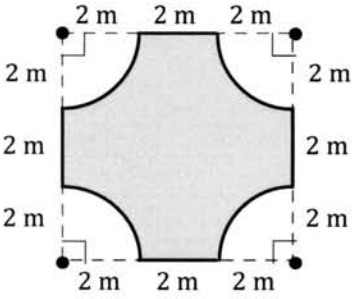


7.



Give perimeter to the nearest mm and area in cm^2 to nearest cm^2.

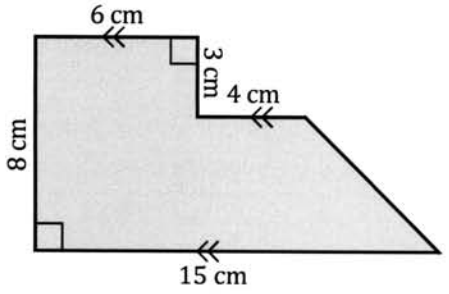
8.



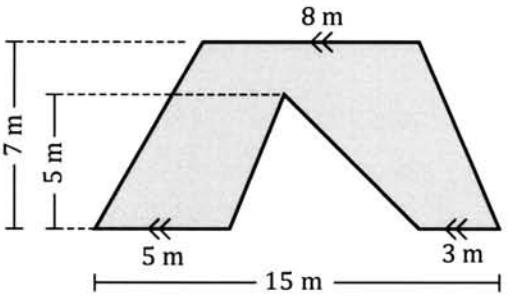
Give perimeter to the nearest cm and area in m^2 rounded to 2 d.p.

For questions 9 to 16 find the area of the shaded region.

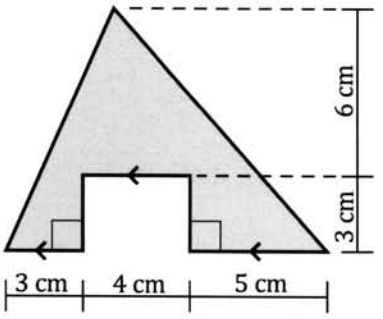
9.



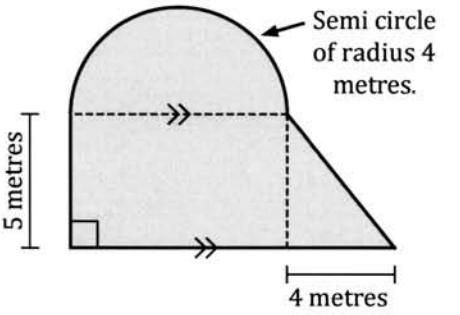
10.



11.

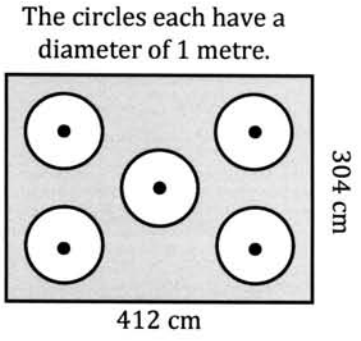


12.



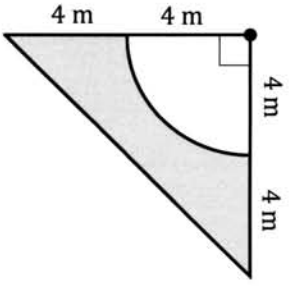
Give area in m^2 rounded to 1 d.p.

13.



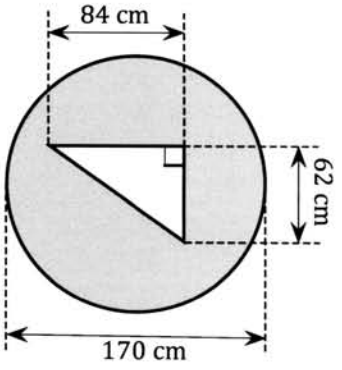
Give area to nearest cm^2.

14.



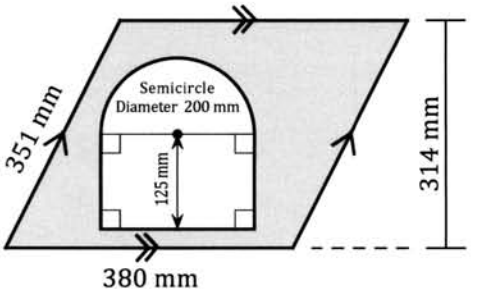
Give area in m^2 rounded to 2 d.p.

15.



Give area to nearest cm^2.

16.



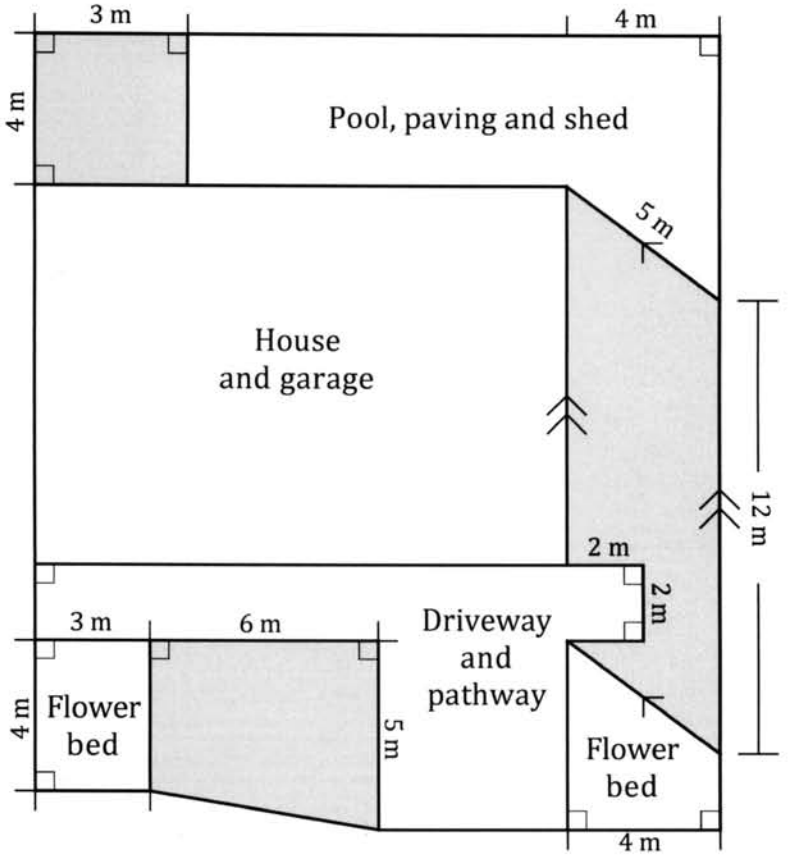
Give area to nearest 10 mm^2.

Applications.

In the following exercise you will again need to determine the perimeter and area of various shapes. However the questions now involve some everyday context for which determining the perimeter or area of a shape is significant.

Exercise 8B.

1. The diagram below shows Bernard's block of land and its various different areas.



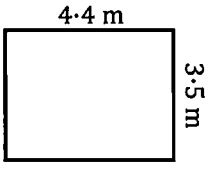
(a) For the three shaded areas Bernard plans to lay lawn using "roll on real grass" which he can purchase for \$11·20 per square metre.
To make sure he has enough lawn Bernard plans to calculate the total area and then round this total up to the next multiple of 5 square metres.
How much will the order for "roll on real grass" cost Bernard?

(b) Alternatively Bernard could have synthetic grass in these areas which he could buy for \$24 per square metre and lay it himself, or for \$37 per square metre he could buy it and have it laid professionally. Again rounding up to the next multiple of 5 m^2 how much would each of these options cost?

(c) Bernard is considering having concrete edging put along all of the lawn area edges that do not form edges with the house and garage area, driveways, pathways, or the property boundary.
Bernard can get this edging done for \$27 per metre.
How much would this edging cost him?

2. "Skirting board" is the decorative strip of wood that runs around the base of the interior walls of a house to cover any gap and uneven edges where the wall meets the floor.

Ignoring any door spaces or other places where skirting board would not be placed, a room with the rectangular floor shape shown on the right, would require 15·8 metres of skirting board (= 4·4 m + 3·5 m + 4·4 m + 3·5 m).

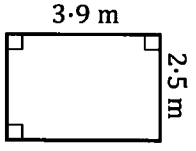


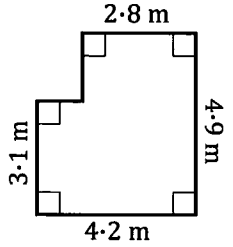
Suppose that skirting board is sold in lengths of 2·4 metres and 3 metres, and that only complete lengths can be purchased.
For the room illustrated we could buy:

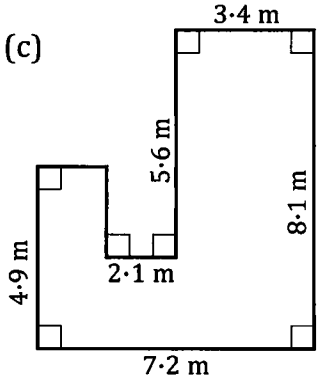
	7 lengths of 2·4 metres	and have	1 metre excess.
Or	6 lengths of 3 metres	and have	2·2 metres excess.

Or, for minimum excess:
3 lengths of 3 metres and 3 lengths of 2·4 metres and have just 0·4 metres excess.

For each of the floor shapes shown below,
→ ignoring any door spaces etc where skirting board would not be placed,
and → assuming that the skirting board can be purchased in just two standard lengths, 2·4 metres and 3 metres,
and → assuming that only complete lengths can be purchased,
determine statements like those shown for the 4·4 by 3·5 m floor plan.
I.e. One statement giving the number of 2·4 metre lengths plus excess.
One statement giving the number of 3 metre lengths plus excess.
And also
One statement giving the combination of the two lengths that will give the minimum excess. (Challenging.)

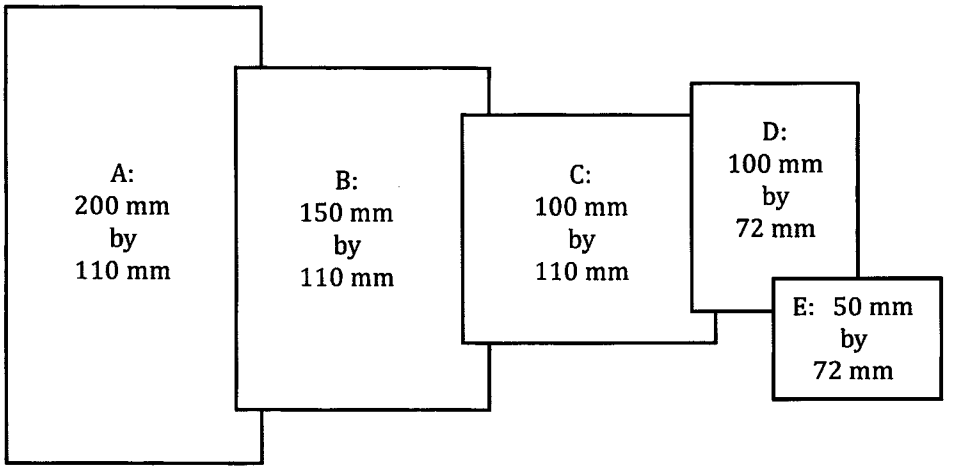
(a)


(b)


(c)


(In reality, some people would choose the lengths to minimize the number of joins, door spaces would not be ignored, some excess would be wanted to allow for joins etc. and some other lengths may well be available for purchase.)

3. A newspaper offers advertising space in five standard sizes, A to E, as indicated below (not drawn to full size).



For each size the cost for an advertisement has a basic price of \$18·50 per square centimetre. The paper keeps some pages free of any advertisements and for other pages it has the following loadings added if particular pages are requested:

Page in paper	Percentage added to basic price.
Page 3	60%
Page 5	45%
Page 7, 9 or 11	30%
Any other requested page	10%

If no particular page is requested the editors will place the advertisement on the page that best suits the newspaper lay out and there will be no percentage loading made to the basic price.

Find the cost of each of the following advertisements with this newspaper.

	Size	Page	Requested or Not requested
(a)	A	5	Requested.
(b)	E	7	Requested.
(c)	B	14	Not requested.
(d)	D	3	Requested.
(e)	D	7	Requested.
(f)	C	13	Requested.
(g)	A	7	Requested.
(h)	E	5	Not requested.

4. A company specializing in applying a protective floor covering to factory floors, warehouse floors, garage floors etc has a special offer, as shown in the advertisement on the right.

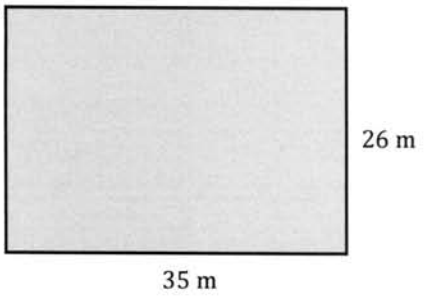
   The company calculates the area of the floor involved, charges \$45/m², reduces this price by 12% during the special offer and then rounds down to whole numbers of dollars.

   Under this special offer what would this company charge for applying the industrial strength floor covering to each of the floor areas shown shaded below?

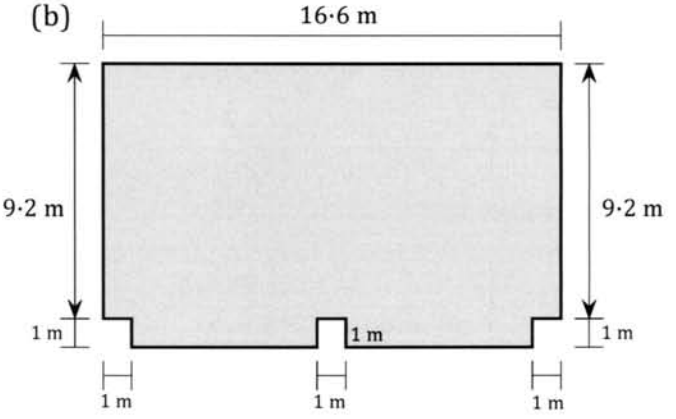
> PROFESSIONALLY APPLIED INDUSTRIAL STRENGTH FLOOR COATING.
>
> **Now 12% off**
>
> of our normal price of \$45 per square metre.
>
> Resistant to chemical staining.
> Resistant to abrasive action.
> Suitable for factories, garages, warehouses, kitchens etc.
>
> Offer available until end of September only.
> Price assumes floor is clear of machinery etc.

Note: ☞ Angles that look right angled in the diagram should be assumed to be right angles.

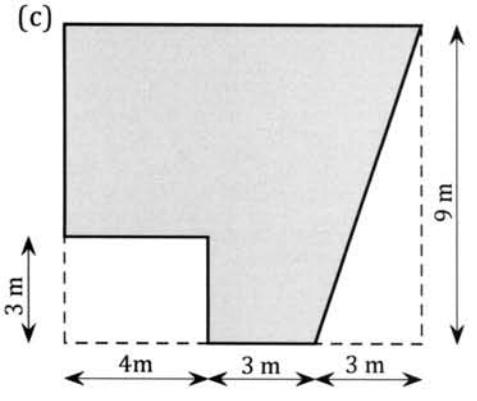
(a)



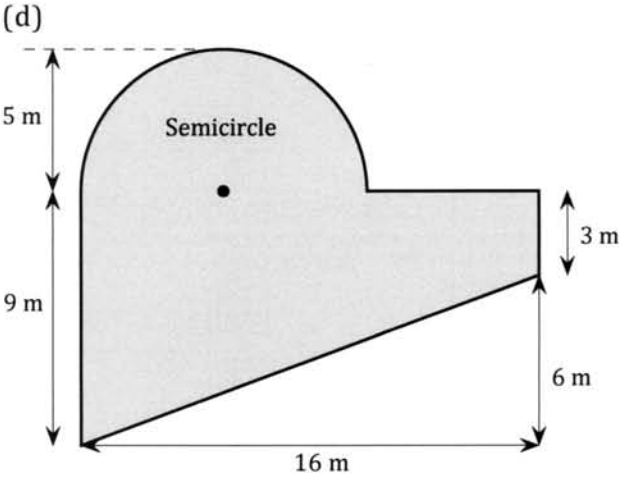
(b)



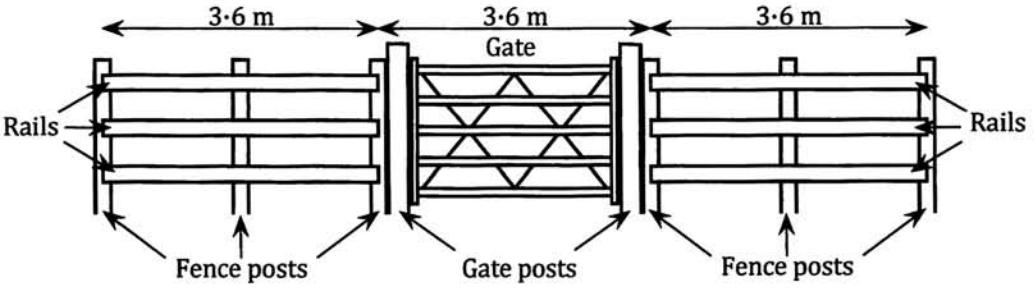
(c)



(d)



5. The diagram below shows a section of a particular type of farm fencing, and gate.
The fencing consists of horizontal wooden fence rails each of 3·6 metres in length, attached to wooden fence posts which are cemented into the ground at 1·8 metre intervals.
The gate can be purchased ready made and, together with its posts, spans the gap between posts as shown below.



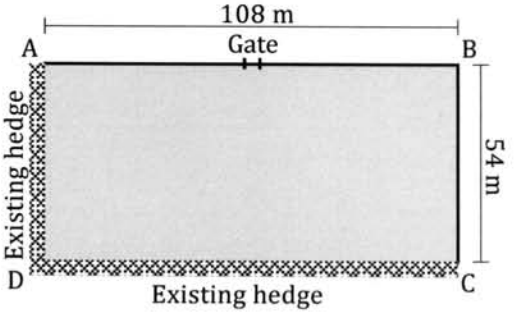
Lengths that are not exact multiples of 3·6 metres can be fenced by cutting rails to an appropriate length.

The order form/invoice for the materials required for the section shown in the diagram above would be as shown below.

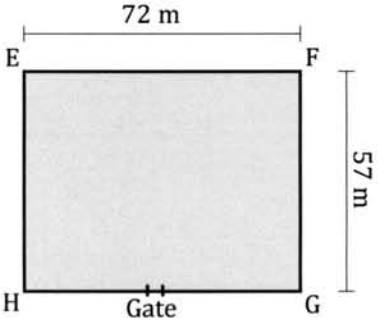
Qty	Item	Unit Price ($)	Total ($)
6	Fence post	10·00	60·00
6	Post pack (Cement, brackets and nails for 1 post)	18·00	108·00
1	Gate pack (Gate, gate posts, latch, cement and all fastenings)	240·00	240·00
6	3·6 metre rail	8·00	48·00
		Sub total	$456·00
		GST (10%)	$45·60
		Grand total	**$501·60**

Complete a similar order form for using this style of fencing for each of the following rectangular paddocks.

(a) New fencing along AB and BC with a gate in AB.

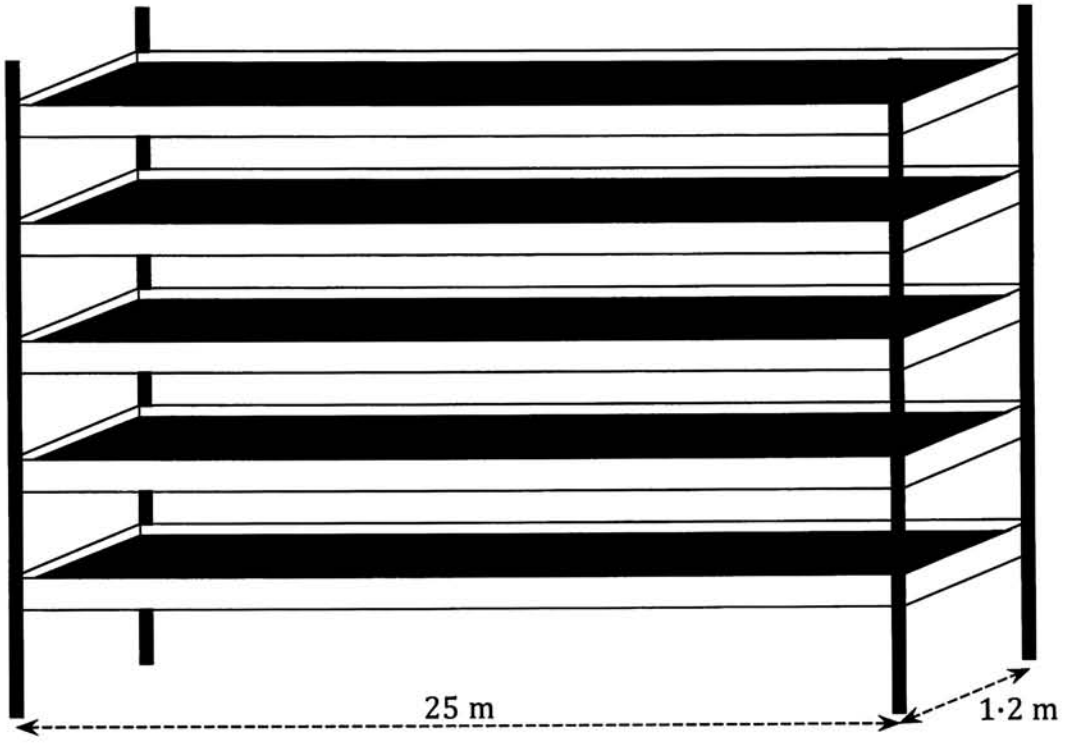


(b) Fence entire perimeter of rectangle EFGH with a gate in HG.



6. For a "6 week picking time" the owner of a mushroom farm expects his farm to produce an average yield of 16 kg of mushrooms for each square metre under cultivation.

What quantity of mushrooms should the farmer expect in one such six week picking time from a system which involves a cultivated area consisting of 18 sets of the 5 tray rectangular stacking structure shown below (not drawn to scale)?



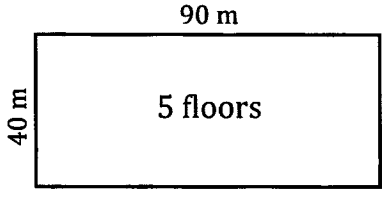
7. A city council is investigating the cost of constructing some multi-storey car parks reasonably close to a proposed sports and entertainment centre. Research suggests that 325 parking spaces can be situated on each hectare (100 m × 100 m) of available space. Thus a high rise parking facility with 5 floors and 0·4 hectares of available space on each floor could accommodate 650 parking spaces.

   Research also suggests that the construction costs for the sort of structures the council is considering cost, on average, $22000 per parking space, once the costs of access, lifts, ramps, lighting, stairs etc have been included.

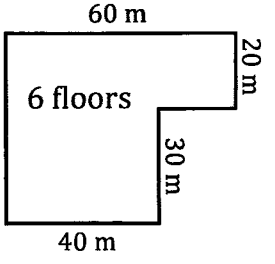
   The council is also advised that once a multi-story car park is built the ongoing maintenance costs to cover, lighting, repairs, security, fee collection etc average $450 per parking space per year. (Whilst this should be more than covered by the parking fees council want to know what the cost is likely to be for planning purposes.)

   Based on these figures what would be the cost of constructing multi-storey car parks for which each floor has the available space shown and the number of floors (including the ground floor) is as indicated? Also determine the annual ongoing maintenance costs for each car park. (Angles that appear to be right angles on the diagrams should be assumed to be right angles.)

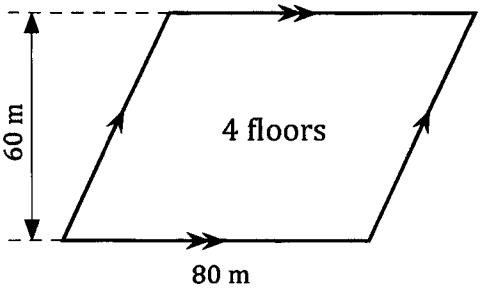
(a)



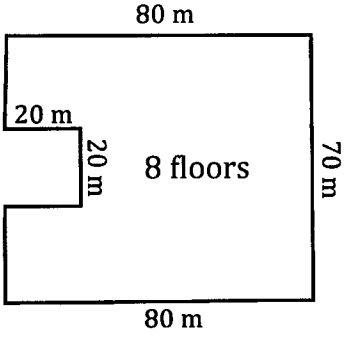
(b)



(c)



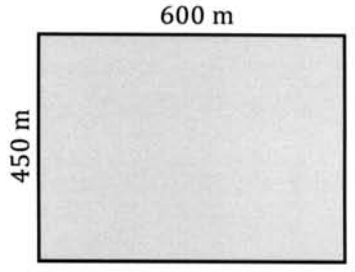
(d)



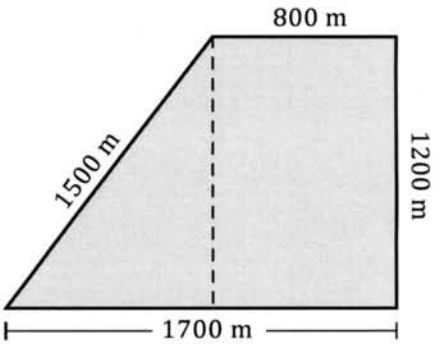
8. In a particular wheat growing region farmers can expect that, with the usual weather pattern, if they sow 80 kg of seed per hectare (100 m × 100 m) they will achieve a wheat yield of approximately 180 grams per square metre provided they properly prepare and maintain the land.

   For this growing region, and based on the figures above, what quantity of seed would each of the following shaded areas require, to the nearest 10 kg, and how much wheat should a farmer expect to harvest from each area (given usual weather patterns and properly prepared and maintained land)?
   Angles that appear right angled in the diagram should be assumed to be right angles.

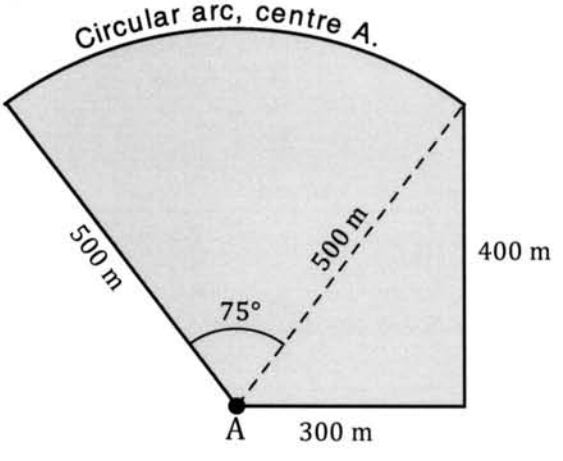
(a)



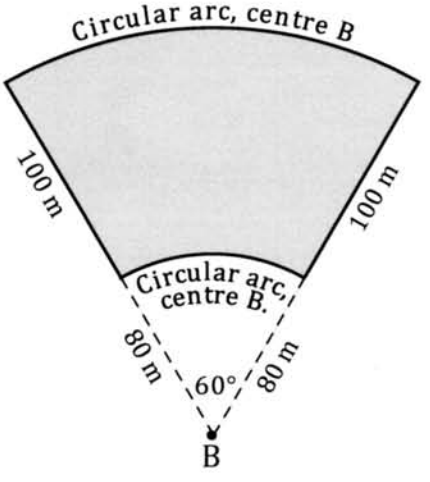
(b)



(c)



(d)



9. For the rate mentioned in the advertisement on the right find the price for having this company clean carpets that cover each of the shaded areas shown below. (Round each answer to the nearest whole dollar.)
Angles that look like right angles in the diagrams should be assumed to be right angles.

Commercial Carpet Cleaning

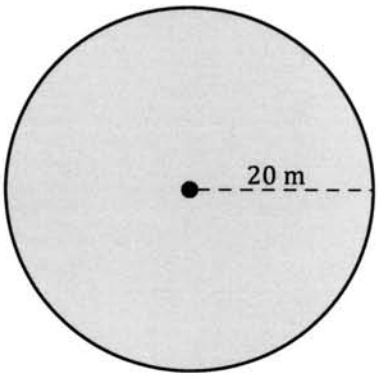
SPECIAL RATES

Only \$0·95 per square metre*

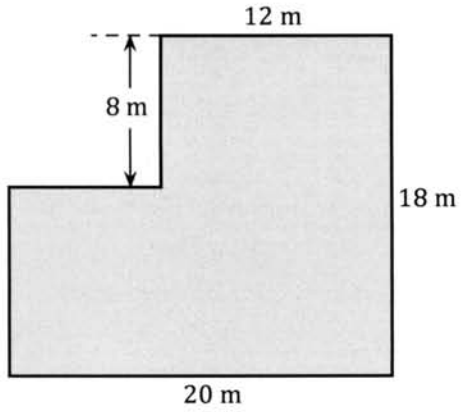
Let us steam clean your commercial carpeted areas.

*For regular contracts

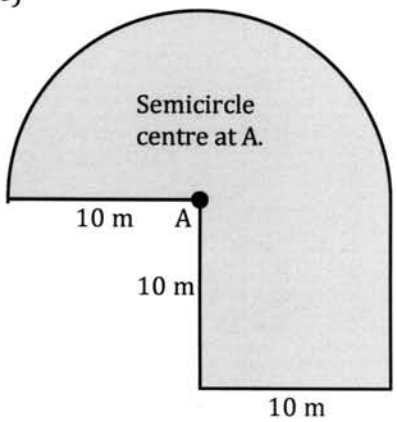
(a)



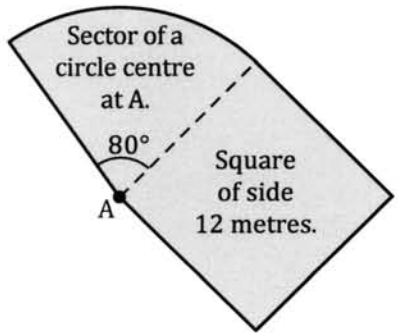
(b)



(c)



(d)

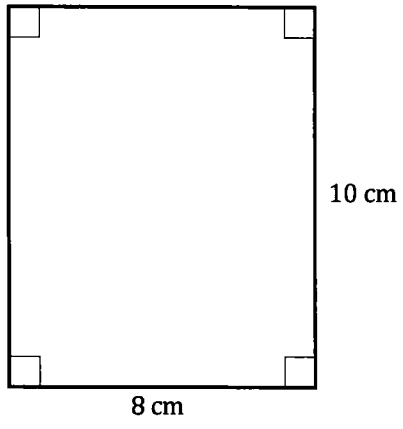


10. Hair density of 80 follicular units per square centimeter is considered normal average hair density.

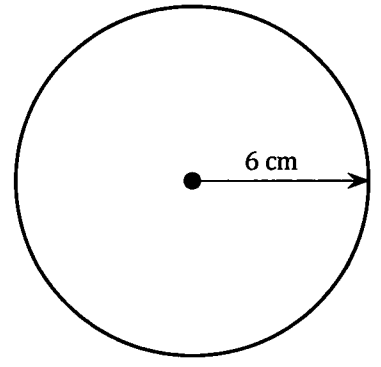
A hair transplant specialist tends to graft 50 follicular units per square centimetre onto bald areas because, although being less than the normal average hair density, the specialist considers the 50 units/cm^2 to be quite sufficient for the graft to appear to be normal density.

At this 50 follicular units/cm^2 how many follicular units will the specialist graft into each of the following bald areas. (Give answers rounded up to the next multiple of 10 follicular units.)

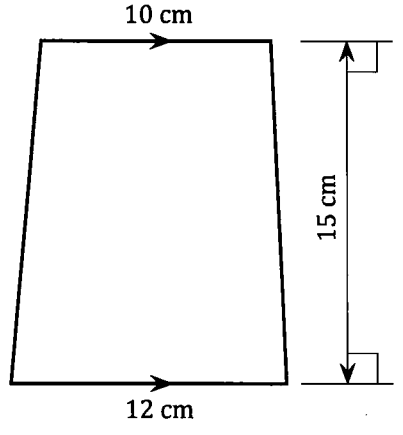
(a)



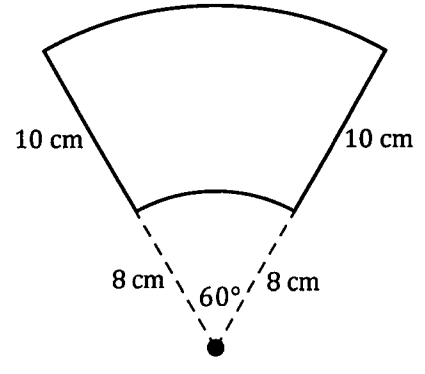
(b)



(c)



(d)



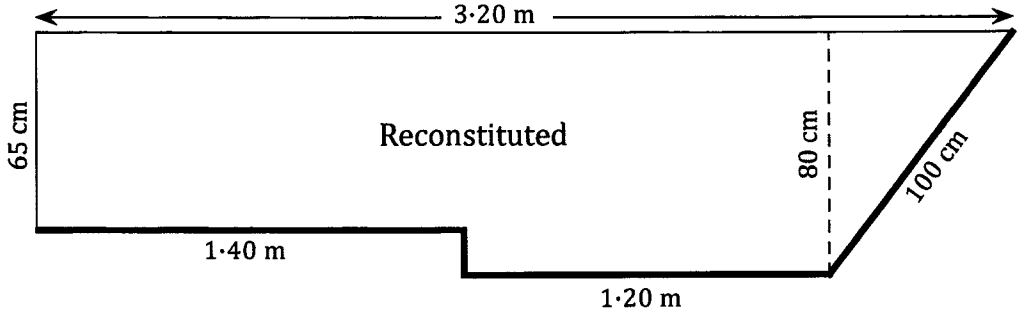
11. A kitchen renovation company offers two types of stone for stone benchtops. The company calculates the price on a cost per square centimetre basis and then rounds to the nearest dollar to give the quote.

Type:	Reconstituted	Cost:	\$0·0384 per cm^2
Type:	Natural	Cost:	\$0·0435 per cm^2

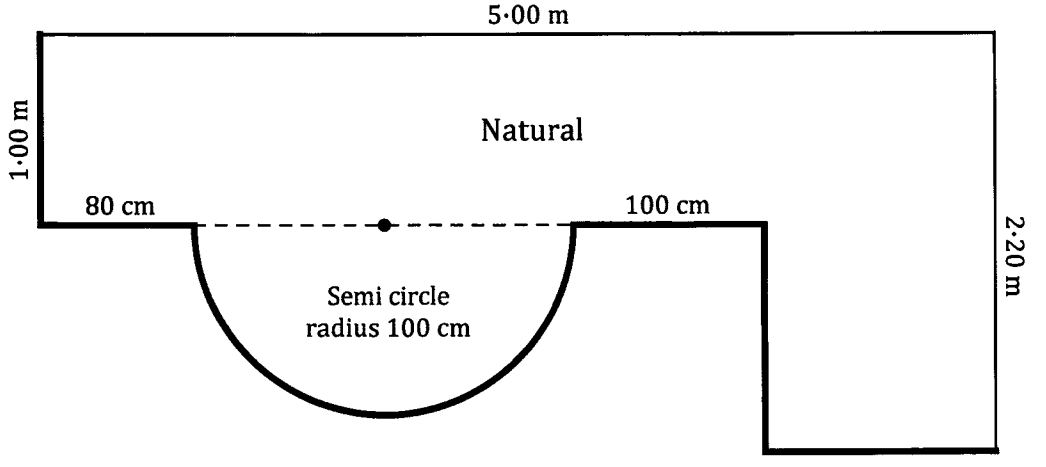
- ☞ No charge is made for cutting out sink or stove spaces but the area is calculated as if these bits were not removed. (The cut out pieces have their edges polished and are given to the customers for use as chopping boards.)
- ☞ Any circular shapes or parts of circles will be charged on their area but then a 30% loading will be added to the cost of the circular area due to wastage.
- ☞ All exposed ends and edges are polished without any additional charge.
- ☞ Joins will be kept to a minimum but some will usually be necessary.
- ☞ 40 mm "bull nose" front edges can be added for an additional cost of \$1·00 per linear centimetre.

1. Why does the cost of the bull nose edging refer to a cost per *linear* metre?
2. What price will be quoted for each of the following stone bench tops?
   Edges that are to have the "bull nose" front edges added are shown below by the bolder lines. (Angles that look right angled in the diagram should be assumed to be right angled.)

(a)



(b)

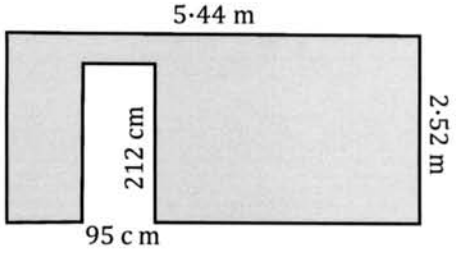


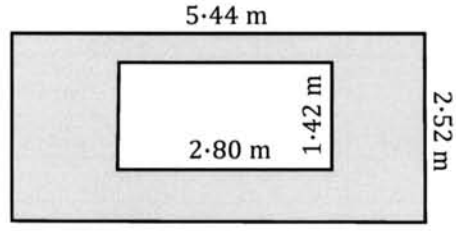
12. A hardware shop sells the *Brush It* brand of quality paint in various sizes of tin.

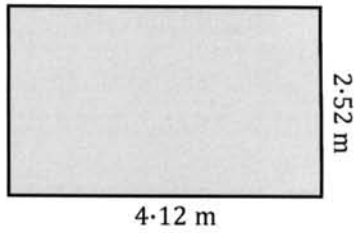
A tin containing	1 litre	of the paint costs	\$42·00.
A tin containing	2 litres	of the paint costs	\$59·90.
A tin containing	4 litres	of the paint costs	\$78·40.
A tin containing	10 litres	of the paint costs	\$179·00.

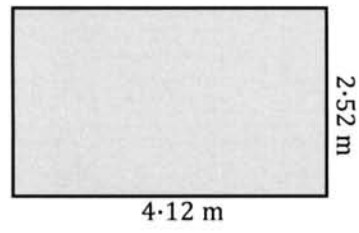
The manufacturer of the paint claims that one litre of paint will put one coat of paint on 16 m^2 of internal wall.

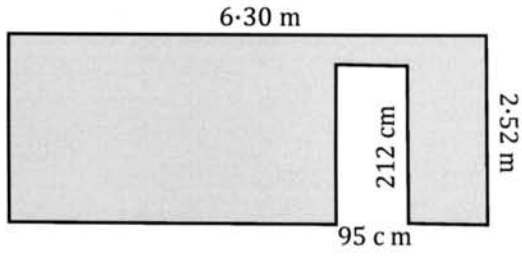
Assuming the manufacturer's claim is correct how many of each size tin of Brush-It should be purchased to put **three** coats of paint on **all** of the following shaded wall areas and what will this paint cost?

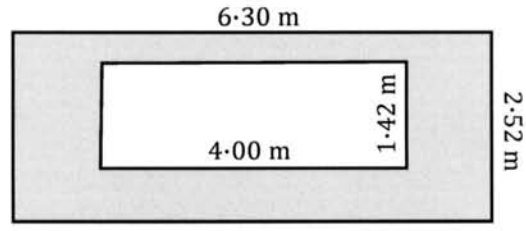


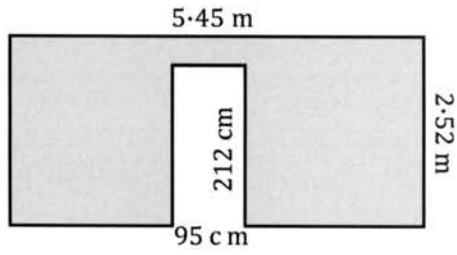


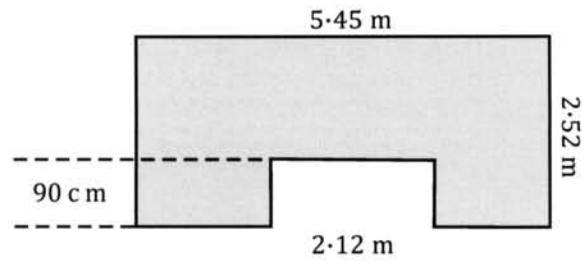












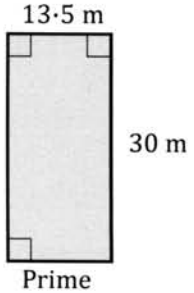
13. A company is developing a number of housing estates in various parts of Australia. Three projects, namely Maynard Waters, Plymptain by Sea and Woodstock Valley, have some identical blocks which, because of location, are priced differently. In these three estates the basic price of the land per square metre is as follows:

Maynard Waters	\$485 per square metre
Plymptain by Sea	\$520 per square metre
Woodstock Valley	\$625 per square metre.

In each estate each block is also graded as standard, premium or prime and these categories then have the above prices increased by 0%, 5% and 15% respectively.

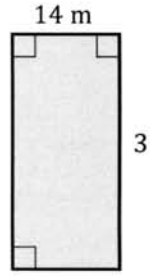
Find the price of each of the following blocks in each of the three estates, giving your answers to the nearest \$1000.

(a)



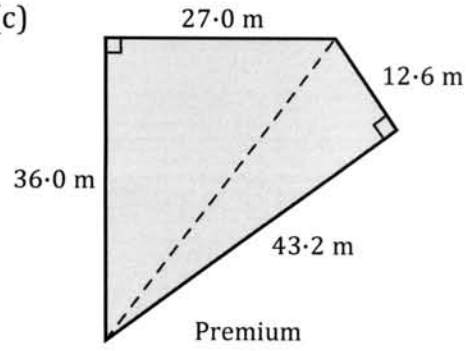
Prime

(b)



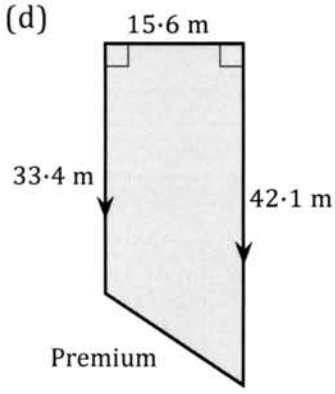
Standard

(c)



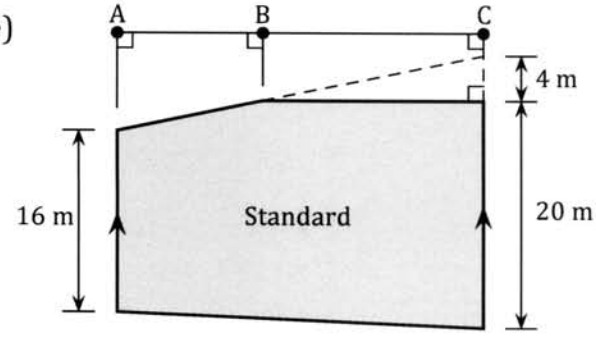
Premium

(d)



Premium

(e)

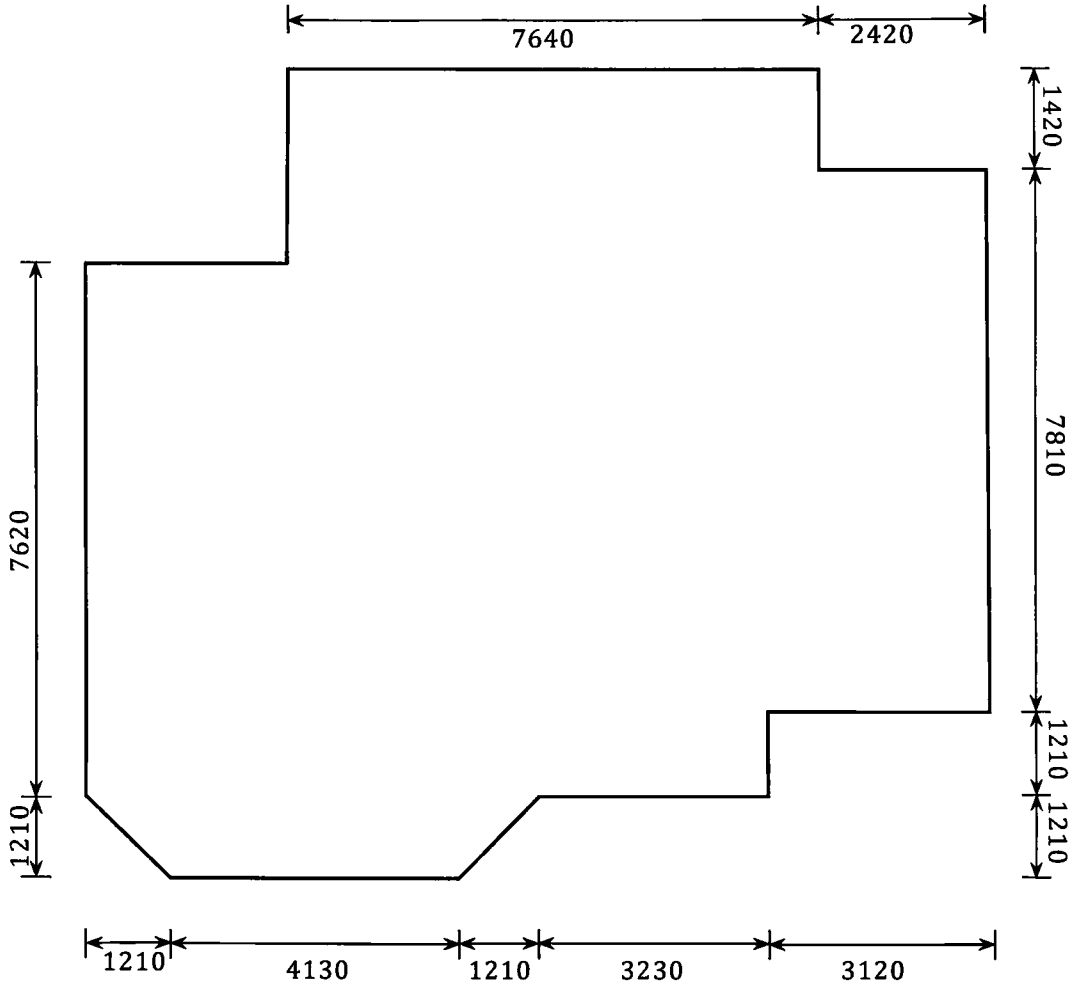


AC = 32 metres
AB : BC = 3 : 5

14. The cost of having a house built depends on the location, the amount of work required to prepare the site for building, the type of construction, the level of finish, the building company and the size of the house.
For a particular building company and region, and with the site ready for building, the following table gives an estimate of the construction cost per square metre of floor plan for a single level dwelling of various types and various finishes

Construction type	Finish		
	Basic	Basic Plus	Deluxe
3 bedroom brick veneer standard design	$1080	$1270	$1620
3 bedroom full brick unique design	$1420	$1670	$2130
4 bedroom brick veneer standard design	$1210	$1420	$1810
4 bedroom full brick unique design	$1590	$1870	$2380

Based on the figures in the table create a similar table for a house with the floor plan as shown below (all measurements are in millimetres) but with your table showing the estimated price for building the house to each type and each finish. (Assume angles in the diagram that appear right angled are indeed right angled.)



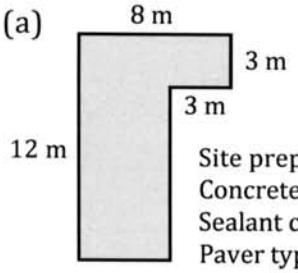
15. For general paving, rather than specialist or ornate work, a company charges \$38·20/m² for site preparation plus an additional \$18·40/m² if existing concrete work needs to be removed. On top of these site preparation costs the company charges \$28·00/m² for laying new pavers. The cost of the pavers is additional to this and the various types of pavers offered by the company are priced as follows:

Sandstone pavers	\$28·95/m²	Limestone pavers	\$37·95/m²
Bluestone pavers	\$42·95/m²	Granite pavers	\$49·95/m²

If the customer wishes, and once the pavers are laid, a sealant coating can be applied for a cost of \$8·50/m²

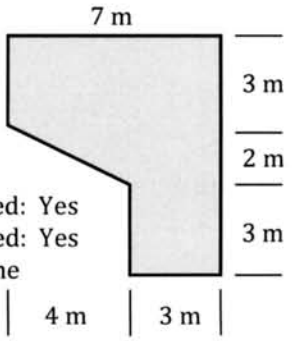
Find the cost of paving each of the areas below using this company to do the paving, with the work and pavers required as indicated. (Angles that look like right angles in the diagram should be assumed to be right angles.)

(a)



Site prep needed: Yes
Concrete removal needed: No
Sealant coating requested: No
Paver type: Sandstone

(b)



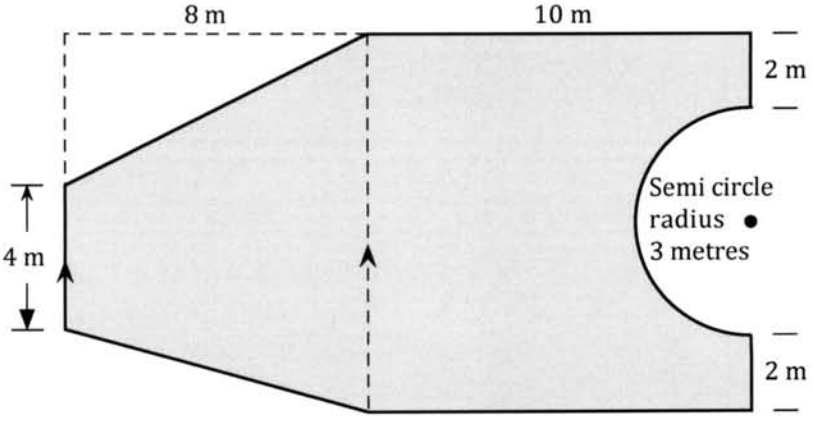
Site prep needed: Yes
Concrete removal needed: Yes
Sealant coating requested: Yes
Paver type: Bluestone

(c) Site prep needed: Yes

Concrete removal needed: Yes

Sealant coating requested: No

Paver type: Limestone



Your turn!

The questions of Exercise 8B have all involved finding the area or perimeter of shapes as applied to situations from real life. Write at least two questions like this of your own invention, researching situations and realistic data from the internet if necessary, and include answers to the questions.

Inverse questions.

The following example, and the exercise that follows, again involve perimeter and area but now you are given one of these quantities and your task is to determine an unknown length. As mentioned in the preface these "inverse questions" do require some equation solving ability and could be regarded as being beyond the requirements of the syllabus for this unit. I leave it to the reader and to teachers to decide whether to cover them or not.

Example 3

If a circle is to have an area of 18 cm^2 what must be the radius of the circle, to the nearest millimetre?

Let the radius of the circle be r cm.

Thus $\pi r^2 = 18$

and so $r^2 = \frac{18}{\pi}$

giving $r = \sqrt{\frac{18}{\pi}}$

$= 2{\cdot}39$ rounded to two decimal places.

The radius of the circle must be 2·4 cm, to the nearest millimetre.

Exercise 8C

1. A square has a perimeter of 64 cm. What is the area of the square?

2. A square has an area of 64 cm^2. What is the perimeter of the square?

3. A rectangle has an area of 36 cm^2. If the rectangle has two sides which are each of length 9 cm what is the perimeter of the rectangle?

4. A square of area 25 m^2 has the same perimeter as a rectangle with a base of length 7 m. Determine the height of the rectangle.

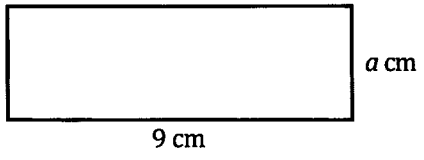
5. If a circle is to have an area of 30 m^2 what should be its radius, to the nearest centimetre?

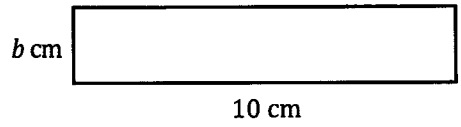
6. A circle has a circumference of 76 cm. Fine the radius of the circle giving your answer to the nearest millimetre.

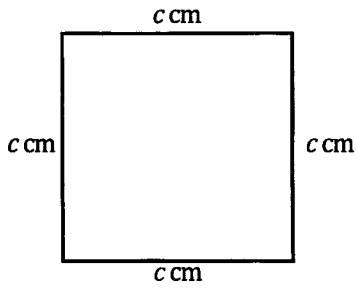
7. If a circle is to have a circumference of 30 m what should be its diameter, to the nearest centimetre?

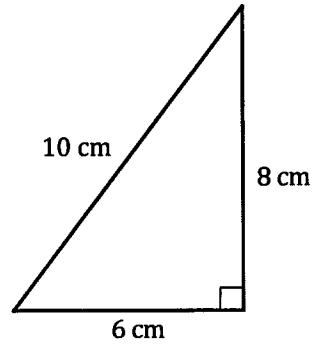
8. A square has the same area as that of a circle of radius 18 cm. Find the length of each side of the square giving your answer in millimetres and to the nearest millimetre.

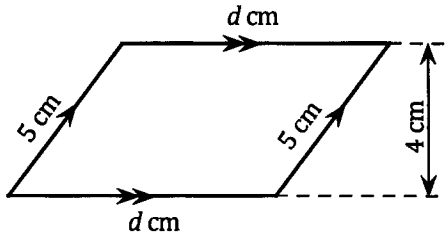
9. Given that each of the following six shapes have the same perimeter find the values of a, b, c, d and e. (Assume that angles that appear right angled in the diagrams are indeed right angled.)
Hence write the areas of the shapes in order, smallest area to largest area.

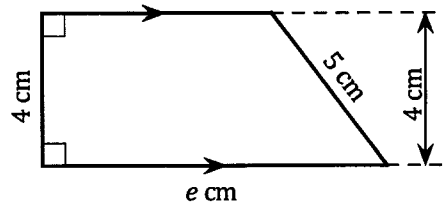




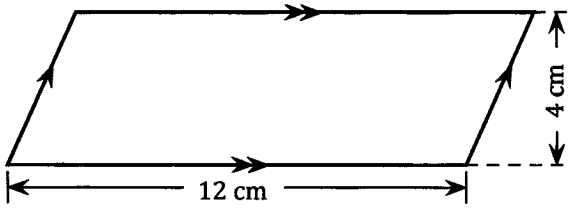


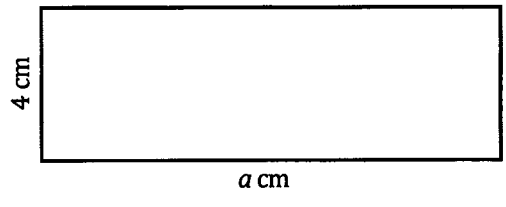


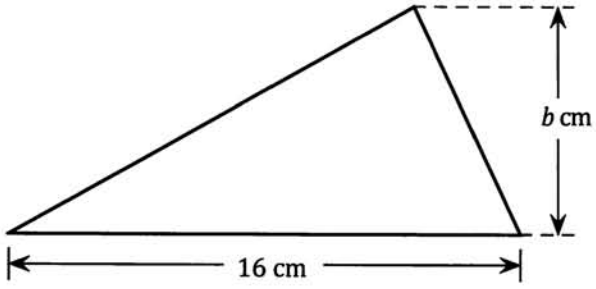


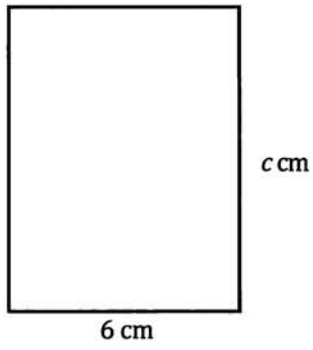


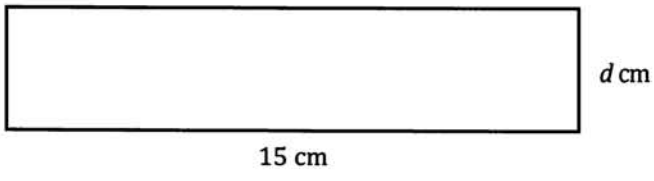
10. Given that each of the following six shapes have the same area find the values of a, b, c, d and e. (Assume that angles that appear right angled in the diagrams are indeed right angled.)

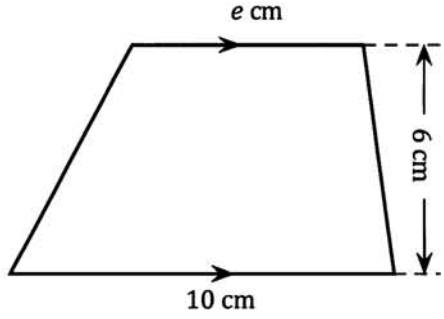




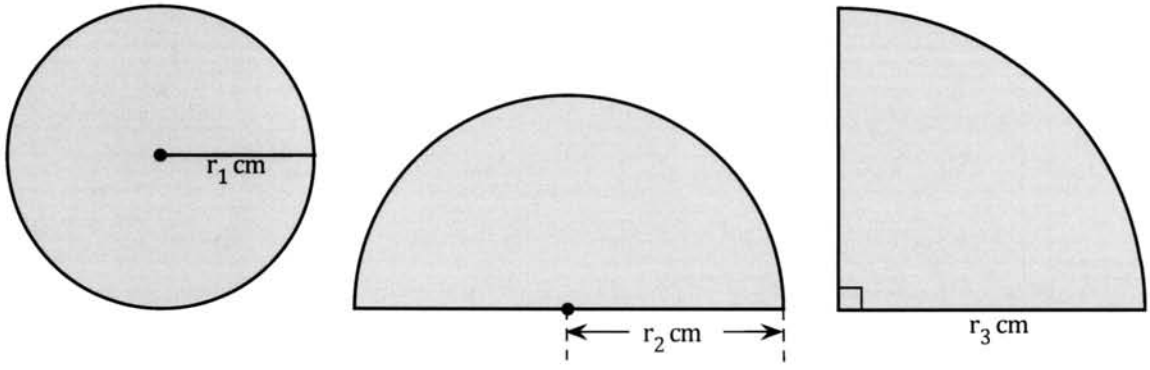








11.



Being told that the circle, the semi-circle and the quarter-circle shown above all have the same area, Jim initially suggests that $r_3 = 4r_1$ and $r_2 = 2r_1$.

However Jim started to doubt this because, thinking the diagrams might be drawn to scale, clearly $r_3 \neq 4r_1$ and $r_2 \neq 2r_1$.

What is the correct relationship between r_3 and r_1 and between r_2 and r_1?

Miscellaneous Exercise Eight.

This miscellaneous exercise may include questions involving the work of this chapter, the work of any previous chapters, and the ideas mentioned in the preliminary section at the beginning of the book.

1. A Government allowance pays \$17500 per year reducing by 60 cents for each dollar earned over \$35000 per year. What would this allowance pay per year to a person who is eligible for the allowance but who earns
   (a) \$30000 per year,
   (b) \$38000 per year,
   (c) \$55000 per year.

2. If $A = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -2 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix}$ find each of the following matrix products stating clearly if any cannot be determined.
   (a) CA (b) BC (c) AB (d) BA

3. Using an exchange rate of \$1 Australian = 0·6572 British Pounds (£) find:
   (a) How many British Pounds can be bought for \$2000 Australian. (Nearest £.)
   (b) The cost in Australian dollars when an item costing £560 is ordered on the internet using an Australian credit card, if an extra \$35 Australian is added in fees for the transfer and transaction.

4. After receiving a discount of 15% off of the normal price Jack is charged \$105·40 for an item. How much would the item have cost him if instead he had received a discount of 20% off of the normal price?

5. Which is the "better buy", and why:
   250 g of sliced ham for \$4·35 or 450 g of the same sliced ham for \$7·95 ?

6. A right triangle has one side of length 6 cm and another of length 8 cm. What could be the length of the third side?

7. How many times would each of the following wheels rotate in a journey of 10 km?
   (a)
   
   Diameter 66 cm
   (b)
   
   Diameter 52 cm
   (c)
   
   Diameter 74 cm

8. A ladder has a length of 7 metres. How far from the base of a wall of height 6·5 m should the base of the ladder be placed if the top of the ladder is to just reach the top of the wall. (Assume the ground is horizontal and the wall is vertical.)

9. If we neglect air resistance then when something is initially held at rest and released, the distance, d metres, that it has fallen t seconds later is given by:

$$d = 4{\cdot}9\,t^2$$

from which it follows that the time taken, t seconds, to fall a distance d metres is given by

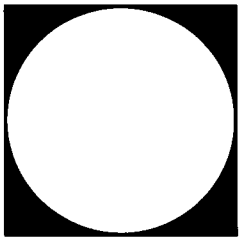
$$t = \sqrt{\frac{d}{4{\cdot}9}}\ .$$

(a) A coin dropped from the top of a building reaches the ground 2 seconds later. How high is the building?

(b) A balloonist inadvertently drops his sandwich from his stationary balloon and sees it hit the ground 6 seconds later. How high was the balloon when the sandwich was dropped (to the nearest ten metres)?

(c) An elderly person leans out of the window of a high rise building to admire the view and her false teeth fall out! How long will they take to reach the ground if the window was 60 metres above the ground?

10. What percentage of the shape shown on the right is shaded? Give your answer to the nearest 0·1%.
(The shape consists of a circle drawn in a square with the circle just touching all four sides of the square.)



11. A company makes three models, A B and C, of a particular item. Each model requires a certain number of units of commodities P, Q and R. Matrix X below shows the number of units of each commodity required to make one of each model.

$$\begin{array}{c} \\ \text{Model A} \\ \text{Model B} \\ \text{Model C} \end{array} \begin{array}{c} \begin{array}{ccc} \text{P} & \text{Q} & \text{R} \end{array} \\ \begin{bmatrix} 2 & 2 & 1 \\ 3 & 1 & 1 \\ 1 & 3 & 1 \end{bmatrix} \end{array} = \text{X}$$

Each unit of P, Q and R costs the company \$50, \$60 and \$200 respectively.

We could write this as a column matrix, Y: $\begin{bmatrix} 50 \\ 60 \\ 200 \end{bmatrix}$

or as a row matrix, Z: $\begin{bmatrix} 50 & 60 & 200 \end{bmatrix}$

Both XY and ZX could be formed but only one of these will contain information likely to be useful.

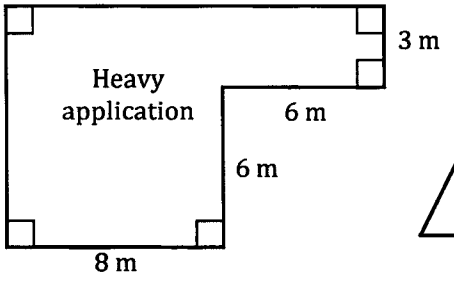
(a) Which is the useful one?

(b) Form the product.

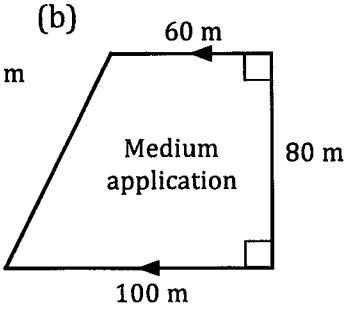
(c) Explain the information it displays.

12. Soil conditioner can be added to the soil to improve the nutrient level in the soil and hence aid the growth of anything planted in the soil. A particular brand of soil conditioner is available in 25 kg bags.
The manufacturer advises that the appropriate application rate of this conditioner depends on whether the existing soil requires minimal application, medium application or heavy application.
The advised rates of application for each classification are as follows:
Minimal application: 100 g per square metre
Medium application: 300 g per square metre
Heavy application: 500 g per square metre.
For each of the following areas, how many bags (rounded up to next whole number) would be needed to add the conditioner at the manufacturer's advised application rate with the application classifications as indicated.

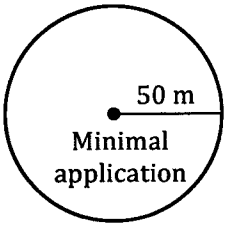
(a)



(b)



(c)



13. The pay slip shown below was created on a spreadsheet. The pay clerk simply enters the bold and italicized entries, i.e. name, normal hourly rate, week and hours and the "boxed amounts" are automatically completed. Use a spreadsheet to produce and print out this pay slip yourself and then generate similar pay slips for:

Patsy Ling, Week 23, Normal hourly rate $19·20,
Normal hours 35, Time and a half hours 4, Double time hours nil.

Troy Marcesi, Week 23, Normal hourly rate $21·40,
Normal hours 35, Time and a half hours 6, Double time hours 4.

ANGUS SWEENEY					
Normal hourly rate	*$17.60*	/hr	Normal	$17.60	/hr
Week	*23*		Time & half	$26.40	/hr
			Double time	$35.20	/hr
Hours worked			Payment due		
Normal	*35*		$616.00		
Time and a half	*3*		$79.20		
Double time	*3*		$105.60		
		Total	**$800.80**		