Number and algebra

6 Expressions and equations

9780170194730

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Patterns and algebra
Introduce the concept of variables as a way of representing numbers using letters.
Create algebraic expressions and evaluate them by substituting a given value for each variable.
Extend and apply the laws and properties of arithmetic to algebraic terms and expressions.
Linear and non-linear relationships
Solve simple linear equations.

Weblink
Algebra masterclass

Algebra is like a written language and it is used to write down ideas in mathematics. To fully understand mathematics, you need to learn to read and write in algebraic ways, just like you learnt to read and write in primary school.

Learning algebra is like learning the alphabet. You need to know the basics before you can start using it properly.

When you work out the area of a rectangle by multiplying the length and width, you are already using algebra. The formula $A = l \times w$ is an example of algebra.

Mathematical literacy

Maths dictionary

The mathematical words below have special meanings that you will learn in this chapter. It is important that you learn to spell them and gradually learn what they mean in mathematics. You may find the glossary or online mathematical dictionary useful for this purpose.

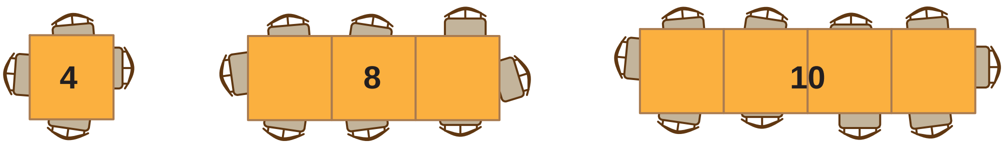
associative	flow chart	right-hand side
coefficient	input	simplify
commutative	inverse operation	solution
constant	law	solve
distributive	left-hand side	substitution
equation	like terms	term
evaluate	linear	unknown
expression	output	variable

6.1 Patterns and symbols

Teacher notes
Restaurant tables

Investigate: Restaurant tables

Many restaurants use square tables that can seat four people. When they get a booking for more than four people, they put tables together so that more people can sit at them. Some examples are shown below. Your teacher might have some cut-outs for you to use in this investigation. Work in small groups.



- How many tables do you need to put together for 6 people?
- How many people can sit at 6 tables put together?
- How many tables will need to be put together for a big birthday party of 20 people?
- Write a rule for the number of people that can sit at any number of tables.

You can see that for 1, 2, 3, 4, … tables, the number of people that can be seated in the investigation above are 4, 6, 8, 10, … If you just look at the number pattern 4, 6, 8, 10, …, the 4 is called the **first term**, the 6 is the **second term**, the 8 is the **third term**, and so on. Each number in the pattern is called a **term** and the **term number** tells you which one it is.

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If you can write a **rule** for the number pattern, it gives you a way to work out other numbers in the pattern. If you can connect the term number to the value of the term, it is easy to work out any term without having to write out the pattern from the start.

Example 1

For the number pattern 3, 4, 5, 6, 7, …
a write a rule for the pattern
b work out the 9th term.

Solution

Method 1

a The numbers go up by 1 each time. — **Start from 3 and add 1 each time to get the next number.**

b Write out the pattern up to 9 terms. — **3, 4, 5, 6, 7, 8, 9, 10, 11, …**

Write the answer. — **The 9th term is 11.**

Method 2

a The 1st term is 3, the 2nd term is 4, the 3rd term is 5, the 4th term is 6, etc., so each term is 2 more than the term number. — **Add 2 to the term number.**

b Work out the 9th term using the rule. — **The 9th term is $9 + 2 = 11$.**

Technology
Excel spreadsheet: Using a rule
MAT07NACT00028

Puzzle sheet
Find the number
MAT07NAPS00049

In Example **1** we could also write the rule using Method 2 as 'the □th term is □ + 2'. For any term, we just fill in the box with the term number. You can also write the numbers in a table. We call the top row of numbers the **input** numbers and the bottom row the **output** numbers, as shown below.

Input	1	2	3	4	5		
Output	3	4	5	6	7		

Example 2

The table below shows input and output numbers for a rule. Write a rule for the output numbers.

Input	5	6	7	8	9	10
Output	15	18	21	24	27	30

Solution

Look for a pattern. — **15 is 3 times 5**

18 is 3 times 6

Write the rule. — **The output number is three times the input number.**

TLF Learning object
Function machine (L3527)

Example 3

For the number pattern 3, 6, 9, 12, …

a write a rule using a box (□) for the term number

b work out the 8th term.

Solution

a Write down how the pattern works. — **The terms go up by 3 each time.**

Write the rule in words. — **Each term is three times the term number.**

Replace the words with symbols. — **The terms are 3 × □.**

b Use the rule. — **The 8th term** $= 3 \times \square$

$= 3 \times 8$

$= 24$

In algebra, instead of using a box, we usually use a letter to represent a number. Then we substitute (put in) the number we want in place of the letter.

Important!

Variables and number patterns

A **variable** (**pronumeral** or **unknown**) is a letter or symbol that stands for a number.
We often use the symbol n for the term number of a number pattern. The symbol a is often used for the term itself, so a_4 is the fourth term of a number pattern.

Example 4

Scientific calculator exercises

Expressions and equations

For the number pattern 5, 10, 15, 20, …, use a variable to:

a write a rule for the pattern

b work out the 9th term.

Solution

a Write down how the pattern works. — **The terms go up by 5 each time.**

Write the rule in words. — **Each term is 5 times the term number.**

Use the symbol n for the term number. — **Each term is 5 ×** n.

b Work out the 9th term. — **The 9th term,** $a_9 = 5 \times 9$

$= 45$

Important!

Linear patterns

Linear number patterns go up or down by the same amount each time. You can write the rule using this amount multiplied by the term number. It is added or taken from the same fixed number each time.

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Example 5

For the number pattern 5, 8, 11, 14, …, use a variable to:

a write a rule for the pattern
b work out the 12th term
c find which term is equal to 32.

Solution

a Check if the pattern is linear. — **The terms go up by 3 each time, so it is linear.**

Write the multiplying part of the rule. — **The rule involves $3 \times n$.**

Write the rule with ? for the fixed number. — **The rule is $? + 3 \times n$.**

Try the rule for the first term. — $? + 3 \times 1 = 5$

Work out the missing number. — **? must be 2.**

Write the rule. — **The terms are $a_n = 2 + 3 \times n$.**

b Work out the 12th term.

$$\begin{aligned} a_{12} &= 2 + 3 \times 12 \\ &= 2 + 36 \\ &= 38 \end{aligned}$$

c Try some numbers out to find the answer. — $2 + 3 \times 9 = 2 + 27 = 29$

Write the answer. — $2 + 3 \times 10 = 2 + 30 = 32$
32 is the 10th term.

Worksheet
Writing a rule
MAT07NAWK00001

Puzzle sheet
Finding the term
MAT07NAPS00001

Animated example
Using a variable
MAT07NAAE00007

Example 6

Write a rule in symbols for the following table of inputs and outputs.

Input p	2	3	4	5
Output v	19	16	13	10

Solution

Write down how the pattern works. — **When the input number goes up by 1, the output number goes down by 3.**

Write the rule using ? for the fixed number. — $v = ? - 3 \times p$

Use the first term to work out ?. — $19 = ? - 3 \times 2$

$19 = ? - 6$

Work out ?. — **? must be 25.**

Write down the rule. — $v = 25 - 3 \times p$

TLF Learning object
Exploring linear equations (L6553)

TLF Learning object
Exploring algebra (L6552)

Example 7

A large barbecue costs \$30 for 1 day's hire and \$10 for every extra day after that.

a Write a rule for the cost of hire.
b Find the cost to hire for 3 days.

Solution

a Change the rule so the first day includes \$10. — **The first day costs \$20 plus \$10.**

Now write down the rule. — **It costs \$20 to hire plus \$10 for each day of hire.**

Write the rule for d days. — **Hire cost = \$20 + \$10 × *d***

b Work out the cost for 3 days. — **Cost for 3 days = \$20 + \$10 × 3**
= \$50

Enter as: 20 [+] 10 [×] 3 [=].

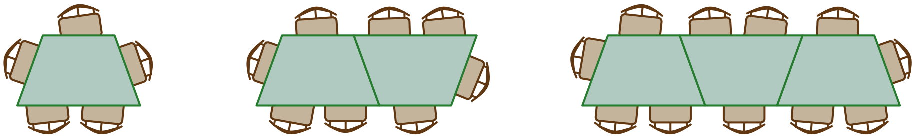
`20+10×3    50`

Exercise 6.1 Patterns and symbols

Understanding

Extra questions
Exercise 6.1
MAT07NAEQ00001

1 A caterer uses portable trapezoidal tables for seating guests at functions. Tables are joined end-to-end to seat the guests as shown here.



a Extend the pattern to complete this table.

Number of tables (t)	1	2	3	4	5	6
Number of guests (g)	5	8				

b Use the information to make a rule in words for the number of guests that can be seated.
c Write the rule using □ for the number of tables
d How many people could be seated at 6 tables?
e How many tables would need to be joined in this arrangement to seat 38 people?

See Example 1

2 Write a rule in words for each of the following number patterns and work out the indicated term:

a 6, 7, 8, 9, 10, …; 15th term
b 4, 8, 12, 16, 20, …; 21st term
c 7, 9, 11, 13, 15, …; 9th term
d 20, 19, 18, 17, …; 11th term
(e) 12, 15, 18, 21, 24, …; 12th term
f 11, 8, 5, 2, $^-1$, …; 10th term

Worked solutions
Exercise 6.1
MAT07NAWS00001

See Example 2

3 Write a rule in words for each of the following number patterns.

a

Input	1	2	3	4	5
Output	5	10	15	20	25

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b

Input	6	7	8	9	10
Output	4	5	6	7	8

c

Input	2	3	4	9	10
Output	8	11	14	29	32

d

Input	3	6	9	12	15
Output	7	13	19	25	31

e

Input	4	7	8	9	12
Output	5	11	13	15	21

f

Input	3	8	10	9	7
Output	13	33	41	37	29

4 For the number pattern 3, 6, 9, 12, …:

See Example 3

a write a rule using a box (□) for the term number

b work out the 8th term.

5 For the number pattern 2, 7, 12, 17, …:

a write a rule using a box (□) for the term number

b work out the 6th term.

6 For the number pattern 7, 11, 15, 19, …:

a write a rule using a box (□) for the term number

b work out the 10th term.

Fluency

See Examples 4, 5

7 For each of the following number patterns, use the variable n to write the rule and find the indicated term.

a 17, 20, 23, 26, 29, …; 10th term

b 1, 3, 5, 7, 9, …; 8th term

c 1, 5, 9, 13, 17, …; 12th term

d 18, 16, 14, 12, 10, …; 20th term

e $^{-}3$, 1, 5, 9, 13, …; 14th term

f 4, 1, $^{-}2$, $^{-}5$, $^{-}8$, …; 9th term

Worked solutions

Exercise 6.1

MAT07NAWS00001

See Example 6

8 Write rules in symbols for the following tables of inputs and outputs.

a

f	1	2	3	4	5	6
h	1	4	7	10	13	16

b

m	1	2	3	4	5	6
p	2	7	12	17	22	27

c

m	0	1	2	3	4	5
b	3	6	9	12	15	18

d

h	3	4	5	6	7	8
k	8	10	12	14	16	18

e

r	0	1	2	3	4	5
s	1	4	7	10	13	16

f

a	2	3	4	5	6	7
b	2	4	6	8	10	12

Problem solving

9 Doreen is saying some numbers and someone hears her say '11, 14, 17, 20, 23'. What will be the 15th number she says?

10 '*One elephant went out to play, upon a spider's web one day*
He found it such tremendous fun, he called three more elephants to come
Four elephants went out to play, upon a spider's web one day
They found it such tremendous fun, they called three more elephants to come
Seven elephants went …
…
They all fell down, and the poem is done!'
The spider's web will not hold more than 20 elephants. How many lines does the poem have?

See Example 7

11 A silo has 2050 kg of cattle feed. Each day, 70 kg is used to supplement the feed of a herd of cattle. How long is it before the silo needs refilling?

Worked solutions
Exercise 6.1
MAT07NAWS00001

12 As a child in a family gets older, they get more pocket money. Peter is 6 and gets 80 cents a week. Pat is 8 and gets \$2 a week. Sue is 11 and gets \$3.80 a week. Cherie is 16. How much does she get each week?

6.2 Number rules

You already know that you can use 'turn-arounds' for multiplying and adding. The product 8×3 has the same answer as 3×8, and $4 + 5$ has the same answer as $5 + 4$. You can always turn around addition and multiplication.

When you multiply three numbers, for example $7 \times 4 \times 5$, it doesn't matter if you do 7×4 first or 4×5 first. You would probably calculate 4×5 first to get $7 \times (4 \times 5) = 7 \times 20 = 140$ because it is easier than doing $(7 \times 4) \times 5 = 28 \times 5 = 140$. When you add three numbers you can always choose whether to start with the front numbers or the back numbers. For $123 + 27 + 48$ you would add the front numbers first to get $150 + 48$, but for $24 + 38 + 52$ you would add the back numbers first to get $24 + 90$.

Mathematicians have special names for turn-around and front-or-back-first rules.

Important!

Commutative and associative laws

Addition and multiplication are both **commutative**.

The order in which the operation is done does not matter, so $a + b = b + a$ and $a \times b = b \times a$ for any numbers.

Addition and multiplication are both **associative**.

When the operation is performed on three numbers, it does not matter which pair is done first, so $a + b + c = (a + b) + c = a + (b + c)$ and $a \times b \times c = (a \times b) \times c = a \times (b \times c)$.

These rules are called **laws** because they work for all numbers.

Although $5 - 5$ can be turned around to $5 - 5$, you cannot turn $5 - 3$ around, so subtraction is not commutative. To be commutative, you must be able to change the order for *any* numbers. Division is not commutative because $6 \div 3 \neq 3 \div 6$.

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Example 8

Calculate $18 \times 25 \times 40$ in your head.

Solution

Calculate 25×40 first as it is easiest.	**Think 25×4 tens $= 100$ tens $= 1000$**
Now calculate 18×1000.	**Think $18 \times 1000 = 18$ thousand**
Write the answer.	**$18 \times 25 \times 40 = 18\ 000$**

When you have many numbers to add together, you can add them in any order because addition is both commutative and associative. It makes sense to choose the order to make tens.

Example 9

Animated example
The associative law
MAT07NAAE00008

Work out $3 + 24 + 22 + 9 + 11 + 17 + 38$.

Solution

Put the units together to make tens.	$3 + 24 + 22 + 9 + 11 + 17 + 38$
Write down what the units add up to.	**The units make 3 tens and there are 4 units left over**
Add the rest of the tens in your head.	**3 and 2 and 2 and 1 and 1 and 3 makes 12 tens**
Write the answer.	**$3 + 9 + 22 + 17 + 24 + 11 + 38 = 124$**

You can calculate 36×24 in your head by multiplying 36 by 20 and then adding 36×4. These are both easy to do using 'double and put on a zero' and 'double double' to give 720 + 144, which is 864. You can do 47×19 by calculating 47×20 and subtracting 47×1 to get 940 – 47, which is 893. Any multiplication can be broken into two parts like these, but you can also do the opposite when two numbers are multiplied by the same number and then added or subtracted.

$28 \times 34 + 28 \times 16$ can be calculated more easily as 28×50. $28 \times 34 = 952$, $28 \times 16 = 448$ and $28 \times 50 = 1400$, which is indeed $952 + 448$. This works for any case, so if you have to add or subtract the product of the same number with some others, you can always perform the addition or subtraction first. Sometimes this is easier, and sometimes it's easier to do it the other way around. Mathematicians also have a special name for this rule.

Important!

Distributive law

The product of the sum (or difference) of two numbers with another number is the same as the sum (or difference) of the products of that number with each one separately. This can be written in symbols as:

$$a \times (b + c) = a \times b + a \times c$$

and $$a \times (b - c) = a \times b - a \times c.$$

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The addition and subtraction cases of the **distributive law** can be summarised as:

$a \times (b \pm c) = a \times b \pm a \times c$, using the symbol $\pm$ for 'plus or minus'.

Example 10

Find a short cut to work out 102×128.

Solution

Look for a break-up.	**Think 102 is 100 plus 2.**
Work out the parts.	**Think 100×128 is 12 800 and 2×128 is 256.**
Add them together.	**Think 12 800 + 256 is 13 056.**
Write the answer.	**$102 \times 128 = 13\,056$**

The distributive law can be used with variables to get rid of brackets. This is called **expanding brackets**. This is what you are really doing when you use a short cut to work out 47×19.

Example 11

Show the short cut to work out 47×19 using the distributive law.

Solution

Write down the problem.	$\mathbf{47 \times 19}$
Show the break-up of 19 in brackets.	$= \mathbf{47 \times (20 - 1)}$
Show the use of the distributive law.	$= \mathbf{47 \times 20 - 47 \times 1}$
Show the answers to the multiplications.	$= \mathbf{940 - 47}$
Write the final answer.	$= \mathbf{893}$

You should write the equals signs underneath each other so that the reasoning for each step is shown clearly, just like it is in Example **11**.
You can expand the brackets of expressions with variables to work out part of the answer, even if you don't know what the variable stands for.

Puzzle sheet
Expandominoes
MAT07NAPS00002

Example 12

Expand the brackets and simplify $5 \times (m + 8)$.

Solution

Write down the problem.	$\mathbf{5 \times (m + 8)}$
Use the distributive law.	$= \mathbf{5 \times m + 5 \times 8}$
Simplify (work out) 5×8.	$= \mathbf{5 \times m + 40}$

In algebra, we often leave out the multiply sign between symbols, so $5 \times (m + 8)$ can be written as $5(m + 8)$ and $5 \times m$ can be written as $5m$. Try using this convention to write out the expansion of $5(m + 8)$ without using multiply signs between symbols.

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Example 13

Expand the brackets and simplify $6(3p - 7)$.

Solution

Write down the problem.	$\mathbf{6(3p - 7)}$
Put in the multiply sign in your head.	**Think** $\mathbf{6 \times (3p - 7)}$
Use the distributive law.	$\mathbf{= 6 \times 3p - 6 \times 7}$
Simplify $6 \times 3p$ and 6×7.	$\mathbf{= 18p - 42}$

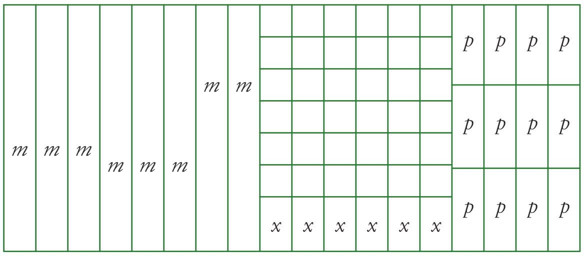
Puzzle sheet
Expand the brackets
MAT07NAPS00003

Investigate: Modelling expansion

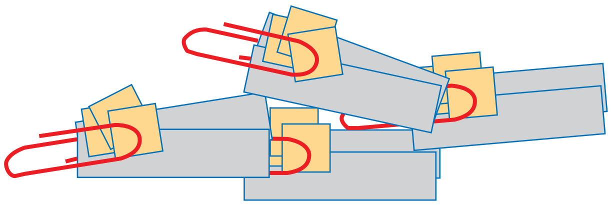
Teacher notes
Modelling expansions

You can use a simple model to show expansion of brackets. You will need some paperclips, small squares and strips of paper coloured on one side to model numbers and variables. You can make the squares and strips as follows:

- Draw lines 1 cm apart on a blank piece of paper in the pattern shown on the right.
- It doesn't matter how long the strips are, as long as they are not an exact centimetre length. Label the strips of equal length m, p and x.
- Use a highlighter or coloured pencil to shade the other side of the paper.
- Cut out the squares and strips and label the reverse sides of the strips.



The white side represents a positive unit and the coloured side represents a negative unit. Use your strips and squares to model $(2x - 3)$ by putting 2 x-strips and 3 coloured-side squares together with a paperclip. Make another 3 models of $(2x - 3)$. Your models should look like this:



Unclip your 4 models to expand $4(2x - 3)$ and collect the x-strips and squares to work out the result of the expansion.
Use your models to work out $3(2 - x)$, $2(4m - 3p)$ and $3(m + 4p - 2x + 5)$.

Puzzle sheet
Expand negative brackets
MAT07NAPS00004

You can model the expansion of $^{-}3(x - 2)$ by making 3 models of $(x - 2)$ and turning them over to change from 3 times to $^{-}3$ times. What do you get as the answer?
Now use your models to work out $^{-}2(5 - 2p)$, $-(2m - 3p + x)$ and $^{-}4(4 - 3p + 2m - x)$.
Write down your results and discuss them in groups of 3 people.

Exercise 6.2 Number rules

Understanding

Extra questions
Exercise 6.2
MAT07NAEQ00002

See Examples 8, 9

1 Work out each of the following in your head and explain how you did it. Your teacher may ask you to explain it to your neighbour.

a $20 \times 154 \times 500$
b $80 \times 136 \times 12.5$
c $75 \times 234 \times 40$
d $68 + 90 + 13 + 43 + 73 + 7 + 37$
e $88 + 52 + 25 + 59 + 24 + 46 + 64$
f $3 + 27 + 74 + 86 + 50 + 92 + 22$
g $79 + 32 + 61 + 22 + 38 + 28 + 20 + 67$
h $88 + 83 + 16 + 77 + 17 + 84 + 20 + 34$
i $72 + 45 + 68 + 11 + 40 + 45 + 67 + 35$

Fluency

See Examples 10,11

Worked solutions
Exercise 6.2
MAT07NAWS00002

2 Use the distributive law to find short cuts for working out each of the following.

a 101×143
b 152×99
c 327×102
d 999×68
e 302×35
f 49×247
g 19×28
h 21×422
i 99×1002

See Example 12

3 Expand the brackets and simplify each of the following.

a $3(a + 2)$
b $2(h + 2)$
c $2(m + 3)$
d $4(x + 6)$
e $4(x - 2)$
f $3(m - 7)$
g $8(k - 3)$
h $5(y - 5)$

See Example 13

Worked solutions
Exercise 6.2
MAT07NAWS00002

4 Expand the brackets and simplify each of the following.

a $4(2m + 3)$
b $7(a + b)$
c $12(2p + 5)$
d $5(a + 2)$
e $6(3x + 4)$
f $12(2m + n)$
g $10(4p + 2q)$
h $3(2a + 4b)$

Problem solving

5 Peter was paid \$12 an hour to pack. He worked 3 hours on Monday, 5 hours on Tuesday, 4 hours on Wednesday and 3 hours on Friday. Explain how to use a mathematical law to work out how much he should be paid.

Worked solutions
Exercise 6.2
MAT07NAWS00002

6 Expand the brackets and simplify each of the following.

a $^{-}3(x + 2)$
b $^{-}7(p + 1)$
c $^{-}2(m - 3)$
d $^{-}5(k - 4)$
e $^{-}4(6 - y)$
f $^{-}9(a + 4)$
g $^{-}(k + 3)$
h $^{-}(m - 2)$
i $^{-}(6 - 2x)$
j $^{-}3(4m + 5)$
k $^{-}5(3y - 6)$
l $^{-}(4 - 7x)$

Reasoning

7 Write down short cuts using the distributive law to work out each of the following. Your teacher may ask you to explain it to your neighbour.

a 103×49
b 999×104
c 205×51
d 1002×458
e 11×734
f 111×75

8 Show how to expand the brackets in each of the following, explaining every step.

a $2(m - p)$
b $7(3f - 2g)$
c $4(3m - 5)$
d $5(3m - 3)$
e $6(3x - 4)$
f $3(1 - k)$
g $6(2 - 3p)$
h $10(2 - 2m)$

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9 Work out a short cut for multiplying by 11 in your head and explain the rule.

10 Work out a rule for multiplying a two-digit number by 99 and explain the rule.

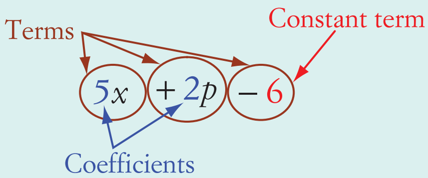
6.3 Algebraic expressions

You have already seen that symbols and letters can be used to represent unknown numbers and that $4 \times a$ is usually written as $4a$. We also usually write $a \times 4$ as $4a$. It follows that $5x + 2p - 6$ means 'multiply 5 by some number, add twice another number and take 6 from the total'. There are special names for the algebraic way of writing this.

Important!

Expressions

An **expression** has variables and/or numbers connected by arithmetic operations like +, ÷ and powers. An expression with numbers only is an **arithmetic expression**, while one with variables is an **algebraic expression**. The numbers are called constants. The numbers that are multiplied by the variables are called **coefficients**, the parts separated from the rest by + or − are called **terms** and a number on its own is called a **constant term**.



To **evaluate** an algebraic expression we put in values for the variables (**substitute** values) and work out the answer.

Video tutorial
Algebraic expressions
MAT07NAVT10011

The sign in front of the term is usually included as part of the term. In the expression above, the terms can be considered as $+5x$, $+2p$ and -6 (that is: $^{+}5x$, $^{+}2p$ and $^{-}6$).

Example 14

What are the coefficients of each of the following?

a $7e$ **b** $^{-}10f$ **c** v **d** $-w$

Solution

a Write the number multiplied by the variable. **The coefficient is 7.**

b Include the sign. **The coefficient is $^{-}$10.**

c There is one v, so $v = 1v$. **The coefficient is 1.**

d Include the sign. **The coefficient is $^{-}$1.**

Example 15

How many terms are there in each of the following expressions?

a $g + 8$ **b** $8 - m + 3pq$ **c** $2(v - 2p)$ **d** $6m + 3(m + 2)$

Solution

a g and 8 are separated by +. **There are 2 terms.**

b The − and + separate the parts. **There are 3 terms.**

c The '$v - 2p$' is not separated from the 2. **There is 1 term.**

d $3(m + 2)$ is one term. **There are 2 terms.**

The English language has many words that mean the same thing, so you need to be able to interpret all the words that describe arithmetic operations.

Important!

Key words for operations

The symbol n is used for a number in the table below. It doesn't make any difference if a different symbol is used for the variable.

Word	Example	Meaning
Sum	The sum of a number and 6.	$n + 6$
Quotient	The quotient of 20 and a number.	$20 \div n$ or $\frac{20}{n}$
Product	The product of a number and negative 8.	$^{-}8n$ or $-8n$
Difference	The difference between 30 and a number.	$30 - n$
Double	A number doubled.	$2n$
Quarter	A quarter of a number.	$n \div 4$ or $\frac{n}{4}$
Increase	A number is increased by 5.	$n + 5$
Less	8 less than a number.	$n - 8$
Decrease	11 is decreased by a number.	$11 - n$

Example 16

Worksheet
Writing algebraic expressions
MAT07NAWK00002

Write each of the following as an algebraic expression, using p and q as variables.

a The difference between a number and 18.
b 21 more than triple a number.
c The product of 8 and the sum of two numbers.
d Substitute the values $p = 3$ and $q = 4$ to evaluate the expression in part **c**.

Solution

a Difference means subtract. $\mathbf{p - 18}$

b Triple means 3 times. $\mathbf{3p + 21}$

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c The total is multiplied by 8. $\mathbf{8(p + q)}$

d Write the expression. $\mathbf{8(p + q)}$

Substitute values. $\mathbf{= 8(3 + 4)}$

Work out the brackets. $\mathbf{= 8 \times 7}$

Multiply. $\mathbf{= 56}$

To check, enter as: 8 (3 + 4) =. | 8(3+4) 56 |

You could substitute $m = 3$ in the **formula** $c = 100m$ to work out the number of centimetres in 3 metres.

One reason for writing algebraic expressions is to make it clear when they can be simplified. The distributive law means that if you have $5p + 8p$ then you can add the 5 and 8 before multiplying by the number.

Important!

Collecting like terms

If an algebraic expression contains terms with exactly the same variables, we call these **like terms**. Like terms can be added or subtracted to **simplify** the expression.

Example 17

Puzzle sheet
Grouping like terms
MAT07NAPS00005

Simplify each of the following by collecting like terms.

a $7m + 8m$

b $9p - 3p$

Solution

a The terms are alike, so you can add them. $\mathbf{7m + 8m}$

$7 + 8 = 15$ $\mathbf{= 15m}$

b The terms are alike, so you can take one away from the other. $\mathbf{9p - 3p}$

$9 - 3 = 6$ $\mathbf{= 6p}$

Example 18

Animated example
Collecting like terms
MAT07NAAE00009

Simplify each of the following by collecting like terms.

a $5x + 3y - 7x + 6y$

b $g^2 + 7gh + 2g - gh$

Solution

a Identify the like terms. $\mathbf{5x + 3y - 7x + 6y}$

Simplify $5x - 7x$ and $3y + 6y$. $\mathbf{= {}^{-}2x + 9y}$

We write a positive term first if we can. $\mathbf{= {}^{+}9y - 2x}$ or $\mathbf{9y - 2x}$

b Identify the like terms (g^2 and g are different). $g^2 + 7gh + 2g - gh$

Simplify $7gh - gh$. $= g^2 + 6gh + 2g$

When expressions are multiplied or divided, it is usual to simplify them if possible by multiplying or dividing the coefficients.

Example 19

Simplify each of the following.

a $3f \times 5d$ **b** $18p \div 6$ **c** $6x \times 5x^2 \times 2y$ **d** $\frac{15mg^3}{9m^2g}$

Solution

a Rearrange the order using the associative law and put the variables in alphabetical order.

$3f \times 5d$

Multiply the coefficients.

$= 3 \times 5 \times d \times f$

$= 15df$

b Divide the coefficients.

$18p \div 6$

$= 3p$

c Multiply the coefficients.

$6x \times 5x^2 \times 2y$

$= 60x \times x^2 \times y$

Use $x \times x^2 = x^3$.

$= 60x^3y$

d Simplify the 15 and 9 by dividing each by 3, write g^3 as $g \times g^2$ and m^2 as $m \times m$.

$\frac{{}^{5}\cancel{15}mg^3}{{}^{3}\cancel{9}m^2g}$

Cancel the m and the g.

$= \frac{5\cancel{mg} \times g^2}{3\cancel{m} \times m\cancel{g}}$

$= \frac{5g^2}{3m}$

Exercise 6.3 Algebraic expressions

Understanding

Extra questions

Exercise 6.3

MAT07NAEQ00003

See Example 14

1 What are the coefficients of each of the following expressions?

a $5h$ **b** ^-7x **c** $6g$
d $2y$ **e** ^-12ab **f** p
g $-gh$ **h** ^-5t **i** $16g^2$
j e^3 **k** $^-892x^3yz$ **l** $412m$
m $6(3x - 5)$ **n** $-(7x - 8y)$ **o** $7x^2(y + 5z)$
p $(3c - d)^2$ **q** $\frac{2q}{3}$ **r** $\frac{3y}{4}$

See Example 15

2 How many terms are there in each of the following expressions?

a $2k + 1$ **b** $m + 3p + n$
c $4x$ **d** $2w + 7y + v$

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e $4 + 7n$
f $4m - 2k - 7$
g 9
h $3p + 2q$
i $x + y + z$
j $4pq + 6q + 7p$
k $6k - 3km + 2m - m^2$
l $6ef + 4e$
m $6(3x - 5)$
n $3x + 2(x + 2y)$
o $7(3m + n) - 4(m - 2n)$
p $\frac{x - 3}{2}$
q $\frac{1}{2}(2x + 4) + 1 - 3(x + 1)$
r $\frac{3(k^2 - 3m)}{5(k - 3)} + 2k - k^2m$

Worked solutions
Exercise 6.3
MAT07NAWS00003

3 Find the like terms in each of these sets.

a $2my, 3x, 6am, 16x, 4mb$
b $2mw, 3km, 4w, 5mw, 6m, 7aw$
c $8k, 3x, 2w, 12g, 23w$
d $2p, 5mq, 5p, 7q, 7m$
e $x, 3x^2, y, x^2$
f $4mn, 3m, 4, 2nm, mn, 2n$
g $x^2y, 2x, 3y, 4x^2y$
h $7, 2a, 4b, 5p, 9, d, 2$

4 Simplify each of these expressions by adding like terms.

See Example 17

a $2m + 5m$
b $4k + 7k$
c $2ab + 5ab$
d $5mn + 2nm$
e $xy + xy$
f $3abc + 4abc + 2bac$

5 Simplify each of these expressions by subtracting like terms.

a $5m - 3m$
b $8d - 3d$
c $12mk - 7mk$
d $45abc - 12abc$
e $45fg - 13fg$
f $48mn - 29mn$

6 Simplify each of these by adding or subtracting like terms.

a $5m + 2m - 3m$
b $12a - 4a - 5a$
c $3x + 5x + 7x$
d $12f + 15f - 18f$
e $8s^2 + 7s^2 - 6s^2$
f $5mn + 6mn - 4nm$
g $3pq - 2pq + pq$
h $11abc + 3abc - 5abc$
i $8p - 3p - 5p$

Fluency

7 Simplify the following.

Worked solutions
Exercise 6.3
MAT07NAWS00003

a $3x + 4 + 5x + 6$
b $4m - 6 + 4m + 10$
c $2mn + 3f + 5mn + 7f$
d $10k - 4 - 6k + 12$
e $23xy + 23ab + 17xy - 17ab$
f $15r + 15 - 15r - r$

8 Simplify the following.

a $6m + 3m$
b $k + 2 + 3k$
c $12mx + 4xm$
d $2d + 9 + 3d$
e $2f + 3g + 4f + 7g$
f $4y + 3x + 2x + 8y$
g $11p - p$
h $7 + 5q - 3$
i $8abc - 4abc - abc$

9 Simplify each of the following.

See Example 18

a $3x + 8y - 8x + 2y$
b $u - 2w - 5u + 7w$
c $-x^2y + 4xy^2 - 6x + 7x$
d $2p - 8p - 8p + 2q$
e $5abc + 3ab - 2bc - 9abc$
f $-d + 4d^3 + 6d + 2d^2$

10 Simplify each of the following.

See Example 19

a $3m \times 2m$
b $5k \times 11k$
c $12f \times 9f$
d $6g \times 3 \times 4h$
e $10 \times 2m \times 6n$
f $4a \times 3 \times 7a$
g $4x \times 7x \times 3$
h $5m \times 2 \times 3m$
i $20x^2 \div 5$
j $18yz \div 6z$
k $\frac{21p^2q}{7pq}$
l $\frac{9mn}{18m^2}$

Worked solutions
Exercise 6.3
MAT07NAWS00003

m $24pq^3 \div {}^-6q^2$ **n** ${}^-7ef \times {}^-9ef^3$ **o** $\frac{{}^-8kp}{24kp}$

p $\frac{28t^3s}{{}^-12t^2s^3}$ **q** ${}^-15m^2 \times {}^-8m^4$ **r** $dgh \times {}^-4ah^2$

s $\frac{{}^-7t^2m}{{}^-35t^3m^3}$ **t** $25gk \div 15km$

Problem solving

11 Write an expression for each of the following. Use N to represent any number.

See Example 16

a double the number
b half the number
c triple the number
d one-quarter of the number
e one-tenth of the number
f the next consecutive number
g 5 times the number
h the sum of the number and 21
i the difference between the number and 10
j 2 more than the number
k the number increased by 3
l the number times itself
m the square root of the number

12 If A, B and C represent any three numbers, write an expression for:

a the sum of A and B
b the sum of all three numbers A, B and C
c the difference between B and C, where B is greater than C
d the product of A and C
e the product of all three numbers A, B and C
f the quotient of A and B
g the sum of A and B, divided by C
h the quotient of C and B.

13 Evaluate each expression in question **12** by substituting $A = 12$, $B = 4$ and $C = 2$.

14 Write an expression for:

a the sum of 3 and A
b 3 less than B
c 5 added to C
d 8 increased by D
e 3 taken away from E
f X decreased by F
g the sum of A, B and W
h m divided by m
i R to the power of 2
j the sum of A and B, divided by 2.

15 Write an expression for:

a the number of students in a class if there are B boys and G girls
b the number of pies needed at a party if there are N children and each child can eat two pies
c the number of children remaining in class if X leave for the library out of a total group of T
d the amount of money earned by selling N cakes at the school fete, where each cake is priced at \$2
e the cost of each film ticket where the total cost is \$$M$ and there are three people going to the film
f the total cost of buying A cans of lemonade and B ice-creams, where each can costs \$1 and each ice-cream costs \$2.

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Reasoning

16 Imagine that you must repeat question **11** using the pronumeral A, instead of N. What difference would this make to your answers? Does it matter which letter of the alphabet you choose to use?

17 Write an expression for the cost of buying c children's tickets at \$21 each and a adult tickets at \$27 each. Select **A**, **B**, **C** or **D**.

A $21a + 27c$
B $a + c$
C $27a + 21c$
D $a + c + 21 + 27$

18 Write formulas for each of the following situations.

a Change M metres to m millimetres.
b Change h hours to d days.
c A curtain has a cut length c, 15 cm more than the finished length l.

19 Substitute the values $M = 1.4$, $h = 72$ and $l = 900$ to evaluate the formulas in question **18**.

20 In the USA, temperature is measured in degrees Fahrenheit. To change degrees Fahrenheit to Celsius, you need to subtract 32, then multiply by 5 and finally divide by 9.

a Write a formula to change f degrees Fahrenheit to c degrees Celsius.
b One mild December afternoon in Miami it was 50°F. Find the temperature in Celsius.
c One summer's day in New York it was 95°F. What was this in Celsius?

6.4 Solving equations

Video tutorial
Solving equations
MAT07NAVT00006

Weblink
Equations in reverse

The equals sign, =, is used in different ways in mathematics. $5 + 4 = ?$ means that we have to work out the answer to the sum $5 + 4$. But $3 \times 4 = 5 + 7$ is a mathematical sentence that says that the product of 3 and 4 is the same as the result of adding 5 and 7. The equation $7 - 3 = 2 \times 3$ is another mathematical sentence. One of these mathematical sentences is true and the other one is false. Which one is true and which one is false?

Important!

Equations

An **equation** is a mathematical sentence. It has an equals sign with expressions on both sides. The expressions can be arithmetic or algebraic.

Example 20

Write equations to show each of the following mathematical sentences and state whether or not the equation is true.

a Five times four is the same as six times three.
b The sum of eleven and eight is the same as the sum of twelve and seven.
c Five times a number is the same as thirty.

Solution

a Write the expressions with an equals sign between them. — $\mathbf{5 \times 4 = 6 \times 3}$

Work out each side. — **Left-hand side** $\mathbf{= 5 \times 4 = 20}$

Right-hand side $\mathbf{= 6 \times 3 = 18}$

The LHS and RHS are not the same. — **The equation** $\mathbf{5 \times 4 = 6 \times 3}$ **is not true.**

b Write the expressions with an equals sign between them. — $\mathbf{11 + 8 = 12 + 7}$

Work out each side. — **Left-hand side** $\mathbf{= 11 + 8 = 19}$

Right-hand side $\mathbf{= 12 + 7 = 19}$

The LHS and RHS are the same. — **The equation** $\mathbf{11 + 8 = 12 + 7}$ **is true.**

c Write the expression, using x as the 'number' and 30 on the RHS. — $\mathbf{5x = 30}$

You don't know what number x is, so you can't say whether or not the equation is true. — **The equation might be true, but it cannot be stated without knowing the number.**

In Example **20**, to work out whether or not the first two equations were true, you worked out the answer to each side. This is called **evaluating** the expressions. You should be able to see that the last equation is true for the number 6, but is not true for any other number. The equation $x^2 - 6x + 8 = 0$ is true for 2 or 4. Some algebraic equations are not true for any numbers, and some are true for more than one number.
An equation can be true, false, or true for some number(s) and false for others.

Important!

Solutions of an equation

A **solution** of an algebraic equation is a value of the variable that makes the equation true.
A solution makes the left-hand side (**LHS**) equal to the right-hand side (**RHS**).
The process of finding the solution to an equation is called **solving** an equation.

Balances like the one below were used for centuries by merchants and traders. If the mass in the left-hand pan is the same as the mass in the right-hand pan, the balance is level. If not, the balance tilts at an angle.

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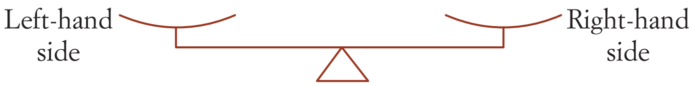
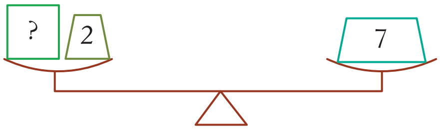
A simple diagram of the balance is shown below.



Example 21

Find the unknown mass if the balance is level.

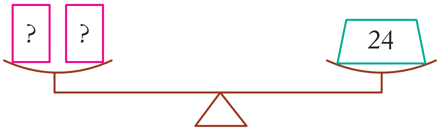


Solution

The left-hand side and the right-hand side are equal. $\boxed{?} + \mathbf{2} = \mathbf{7}$

Think what must be added to 2 to obtain 7. $\boxed{?}$ **must be 5.**

The balance problem below shows two equal masses on the left-hand side.



The balance problem can be written as the following equation.

$$\boxed{?} + \boxed{?} = 24$$

We can also write this equation using a variable as follows.

$$m + m = 24$$
$$\text{or } 2m = 24$$
$$\text{So } m = 12$$

Example 22

State whether the number in the brackets is a solution to the equation.

a $d - 3 = 10$ (7) **b** $f + 8 = 12$ (4)

Solution

a Write down the left-hand side of the equation. **LHS = $d - 3$**

Replace d with 7 and work out the result. **$= 7 - 3 = 4$**

Compare the result with the right-hand side. **LHS = $4 \neq 10$ = RHS**

The left-hand side ≠ right-hand side. **7 is not a solution to $d - 3 = 10$.**

b Write down the left-hand side of the equation. **LHS = $f + 8$**

Replace f with 4 and work out the result. **$= 4 + 8 = 12$**

Compare the result with the right-hand side. **LHS = 12 = RHS**

The left-hand side = right-hand side. **4 is a solution to $f + 8 = 12$.**

One way of solving equations is to guess a solution and check the guess by inserting it into the equation to see if it makes the left-hand side equal the right-hand side.

Example 23

Find a solution to each of the following equations.

a $10 - t = 8$

b $3d = 15$

Solution

a Write down the left-hand side of the equation.	**LHS = 10 − t**
Try $t = 4$ as a solution.	**= 10 − 4 = 6**
$6 \neq 8$.	**4 is not a solution.**
Try $t = 2$ as a solution.	**LHS = 10 − 2 = 8**
State the result.	**t = 2 is a solution to 10 − t = 8.**
b Write down the left-hand side of the equation.	**LHS = 3d**
Try $d = 7$ as a solution.	**= 3 × 7 = 21**
$21 \neq 15$.	**d = 7 is not a solution.**
Try $d = 5$ as a solution.	**3 × 5 = 15**
State the result.	**d = 5 is a solution to 3d = 15.**

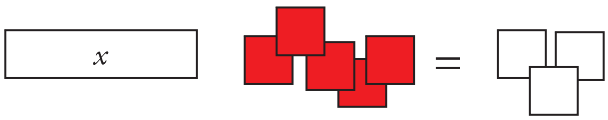
The process used in Example **23** is called the '**guess and check and improve**' or '**trial and error**' method of solving equations.

Investigate: Modelling equations

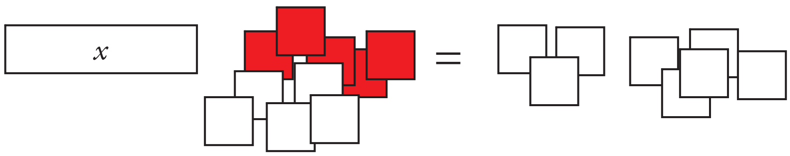
Teacher notes
Modelling equations

Work in pairs for this investigation.

You can use the modelling method from Section 6.2 to solve equations. For this investigation, you will need the strips and squares from that investigation, and a piece of paper with an equals sign in the middle. Use your strips and squares to model the equation $x - 5 = 3$ as shown below.



To get the x on its own, you need to get rid of the 5 negative (red) squares.
Add 5 positive (white) squares to both sides so that it looks like this:



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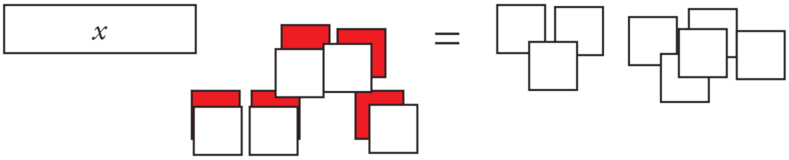
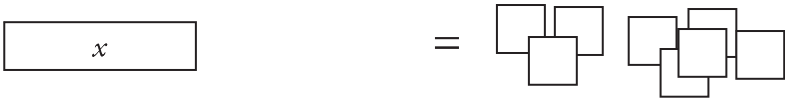
Now put red and white squares in pairs together on the LHS, so that it looks like this:



But each pair means (1 − 1), so each pair of red and white squares comes to nothing. Take them away so the model looks like this:



Counting the squares on the right-hand side gives the answer $x = 8$!
Try using your model to solve the following equations. Remember, to get rid of a number on the left-hand side, add the opposite number to both sides.

$$x + 3 = 7$$
$$p - 5 = 3$$
$$m + 8 = 6$$

Set each other some problems to solve.

There is another way of informally solving equations as well as the balance model, guess and check, and the model in the investigation above. This method uses a **flow chart** that shows the steps involved on the left-hand side of an equation using arrows between a line of boxes. The variable is shown in the first box. The answer from the right-hand side of the equation is shown in the last box. The boxes in-between are for the numbers that result from each operation. The operations are shown above the arrows between the boxes.

Example 24

Write the equation $3d + 5 = 20$ as a flow chart.

Solution

Write the equation.	$\mathbf{3d + 5 = 20}$
Make all operations obvious.	$\mathbf{3 \times d + 5 = 20}$
Show the amounts as boxes and operations as arrows.	$\boxed{d} \xrightarrow{\times 3} \boxed{} \xrightarrow{+5} \boxed{20}$

Animated example
Backtracking
MAT07NAAE00010

A flow chart is used to solve an equation by moving backwards along the boxes (**backtracking**) from the right-hand side to the beginning. At each step, the operation is ‘undone’ by doing the opposite operation. We call the opposite operation the **inverse operation**.

Puzzle sheet
Solving equations
MAT07NAPS00006

Worksheet
Backtracking
MAT07NAWK00003

Example 25

Solve the equation $3d + 5 = 20$ using backtracking on a flow chart.

Solution

Draw the flow chart. $d \xrightarrow{\times 3} \square \xrightarrow{+5} 20$

Put in the inverse operations. $d \underset{\div 3}{\overset{\times 3}{\rightleftarrows}} \square \underset{-5}{\overset{+5}{\rightleftarrows}} 20$

Apply the inverse operations, one step at a time. $\square \xleftarrow[\div 3]{} 15 \xleftarrow[-5]{} 20$

Go all the way to the start. $5 \xleftarrow[\div 3]{} 15 \xleftarrow[-5]{} 20$

State the result. **$d = 5$ is a solution to $3d + 5 = 20$.**

When you find what you think is a solution to an equation, you should always check to see if it works. Do this by putting the value back into the equation. If the LHS = RHS, the value is a solution. In Example **25**, if you substitute 5 in for d on the LHS, you have:

$$\begin{aligned} 3d + 5 &= 3 \times 5 + 5 \\ &= 20 \end{aligned}$$

This means that $d = 5$ is a solution to the equation $3d + 5 = 20$.
Sometimes you will need to swap the left- and right-hand sides and/or make the operations in the equation more obvious before you begin backtracking.

Example 26

Use backtracking to solve the following equations.

a $19 = 4y - 17$

b $30 = 5(2e - 4)$

c $\dfrac{3r - 8}{2} = 11$

Solution

a Swap the LHS and RHS of the equation to get the variable on the LHS. $\mathbf{4y - 17 = 19}$

Make all operations obvious. $\mathbf{4 \times y - 17 = 19}$

Write the equation as a flow chart. $y \xrightarrow{\times 4} \square \xrightarrow{-17} 19$

Apply the inverse operations. $y \underset{\div 4}{\overset{\times 4}{\rightleftarrows}} \square \underset{+17}{\overset{-17}{\rightleftarrows}} 19$

Begin at the final result and backtrack. $9 \xleftarrow[\div 4]{} 36 \xleftarrow[+17]{} 19$

State the result. $\mathbf{y = 9}$

b Swap the LHS and RHS of the equation to get the variable on the LHS. $\mathbf{5(2e - 4) = 30}$

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Show all operations.	$5 \times (2 \times e - 4) = 30$
Express the equation as a flow chart.	$e \xrightarrow{\times 2} \square \xrightarrow{-4} \square \xrightarrow{\times 5} 30$
Apply the inverse operations.	$e \underset{\div 2}{\overset{\times 2}{\rightleftarrows}} \square \underset{+4}{\overset{-4}{\rightleftarrows}} \square \underset{\div 5}{\overset{\times 5}{\rightleftarrows}} 30$
Begin at the result and backtrack.	$5 \xleftarrow[\div 2]{} 10 \xleftarrow[+4]{} 6 \xleftarrow[\div 5]{} 30$
State the result.	$e = 5$

c $\dfrac{3r - 8}{2} = 11$

Show all operations.	$(3 \times r - 8) \div 2 = 11$
Express the rule as a flow chart.	$r \xrightarrow{\times 3} \square \xrightarrow{-8} \square \xrightarrow{\div 2} 11$
Apply the inverse operations.	$r \underset{\div 3}{\overset{\times 3}{\rightleftarrows}} \square \underset{+8}{\overset{-8}{\rightleftarrows}} \square \underset{\times 2}{\overset{\div 2}{\rightleftarrows}} 11$
Begin at the result and backtrack.	$10 \xleftarrow[\div 3]{} 30 \xleftarrow[+8]{} 22 \xleftarrow[\times 2]{} 11$
State the result.	$r = 10$

The modelling method using paper coloured on both sides and the backtracking method both use inverse operations to find the solution to an equation. This is the formal method of solving equations where the left-hand side is a **linear rule** (like $x + 3$ or $6x + 5$, which produce linear number patterns if we input $x = 1, 2, 3, 4, \ldots$).

Important!

Rules for solving equations using inverse operations

1. What is done to one side must be done to the other.
2. To 'undo' or cancel an operation, apply its inverse.
3. Keep the equals signs underneath each other.

Example 27

Solve each of these equations using inverse operations.

a $x + 3 = 12$

b $y - 4 = 8$

Solution

a

Write down the equation.	$x + 3 = 12$
Undo '+ 3' by applying '– 3'.	$x + 3 - 3 = 12 - 3$
Evaluate both sides.	$x + 0 = 9$
State the result.	$x = 9$

TLF Learning object
Algebra balance scales: Negatives (L3510)

b Write down the equation. $y - 4 = 8$

Undo '– 4' by applying '+ 4'. $y - 4 + 4 = 8 + 4$

Evaluate both sides. $y + 0 = 12$

State the result. $y = 12$

Example 28

Scientific calculator exercises
Expressions and equations

Solve each of these equations using inverse operations.

a $2a = 14$

b $\frac{b}{3} = 5$

Solution

a Write down the equation. $2a = 14$

$2a$ means $2 \times a$. We need to undo 'multiplication by 2'.

Undo '× 2' by applying '÷ 2'. $2a \div 2 = 14 \div 2$

Rewrite using fractions. $\frac{2a}{2} = \frac{14}{2}$

Evaluate both sides and state the result. $a = 7$

b Write down the equation. $\frac{b}{3} = 5$

$\frac{b}{3}$ means $b \div 3$. We need to undo 'division by 3'.

Undo '÷ 3' by applying '× 3'. $\frac{b}{3} \times 3 = 5 \times 3$

Evaluate both sides and state the result. $b = 15$

If there is more than one operation, you should do the inverse operations in the opposite order to the normal order of operations.

Example 29

Video tutorial
Two step equations
MAT07NAVT10012

Solve the following equations using inverse operations.

a $6m + 5 = 47$ **b** $16 = 4(5y - 11)$ **c** $\frac{7h - 14}{3} = 14$

Solution

a Write down the equation showing all operations. $6 \times m + 5 = 47$

Undo the '+ 5' by subtracting 5 from both sides. $6 \times m + 5 - 5 = 47 - 5$

Simplify. $6 \times m = 42$

Undo the '× 6' by dividing both sides by 6. $6 \times m \div 6 = 42 \div 6$

Simplify. $m = 7$

To check, enter as: 6 × 7 + 5 =.

6x7+5 47

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b Rearrange the equation. Show all operations.

$$4(5y - 11) = 16$$
$$4 \times (5 \times y - 11) = 16$$

Undo the '× 4' by dividing both sides by 4.

$$\frac{\cancel{4} \times (5 \times y - 11)}{\cancel{4}} = \frac{16}{4}$$

Simplify.

$$5 \times y - 11 = 4$$

Undo the '− 11' by adding 11 to both sides.

$$5 \times y - 11 + 11 = 4 + 11$$

Simplify.

$$5 \times y = 15$$

Undo the '× 5' by dividing both sides by 5.

$$\frac{5 \times y}{5} = \frac{15}{5}$$

Simplify.

$$y = 3$$

To check, enter as: 4 (5 × 3 − 11) = .

4(5×3−11) 16

c Write down the equation showing all operations.

$$\frac{7 \times h - 14}{3} = 14$$

Undo the '÷ 3' by multiplying both sides by 3.

$$\frac{7 \times h - 14}{3} \times 3 = 14 \times 3$$

Simplify.

$$7 \times h - 14 = 42$$

Undo the '− 14' by adding 14 to both sides.

$$7 \times h - 14 + 14 = 42 + 14$$

Simplify.

$$7 \times h = 56$$

Undo the '× 7' by dividing both sides by 7.

$$7 \times h \div 7 = 56 \div 7$$

Simplify.

$$h = 8$$

To check, enter as:
(7 × 8 − 14) ÷ 3 = .

(7×8−14)÷3 14

When equations involve brackets, it is often easier to expand the brackets using the distributive law before solving the equation.

Example 30

Solve the following equations.

a $3(f - 7) = 24$

b $5(2g + 1) + 8 = 33$

Solution

a Write down the equation.

$$3(f - 7) = 24$$

Expand the brackets.

$$3f - 21 = 24$$

Undo the '− 21' by adding 21 to both sides.

$$3f - 21 + 21 = 24 + 21$$

Simplify.

$$3f = 45$$

Undo the '× 3' by dividing both sides by 3.

$$3f \div 3 = 45 \div 3$$

Simplify.

$$f = 15$$

To check, enter as: 3 (15 − 7) = .

3(15−7) 24

b Write down the equation. $5(2g + 1) + 8 = 33$

Expand the brackets. $10g + 5 + 8 = 33$

Simplify. $10g + 13 = 33$

Undo the '13' by subtracting 13 from both sides. $10g + 13 - 13 = 33 - 13$

Simplify. $10g = 20$

Undo the '× 10' by dividing both sides by 10. $10g \div 10 = 20 \div 10$

Simplify. $g = 2$

To check, enter as:
5 (2 × 2 + 1) + 8 = .

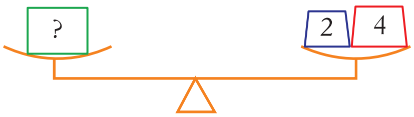
`5(2×2+1)+8    33`

Exercise 6.4 Solving equations

For questions **1** to **6**, the masses shown on the balances are in kilograms. All parcels marked as '?' in any one question are of equal mass. Find the masses of these parcels.

Understanding

Extra questions
Exercise 6.4
MAT07NAEQ00004

1 a


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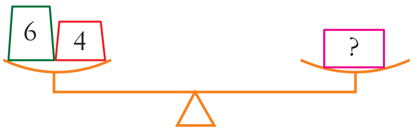

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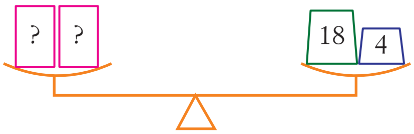

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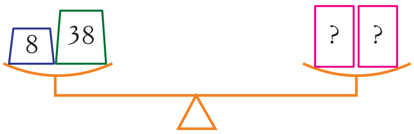

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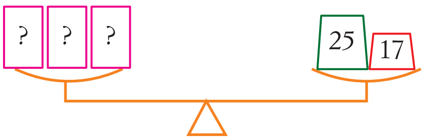

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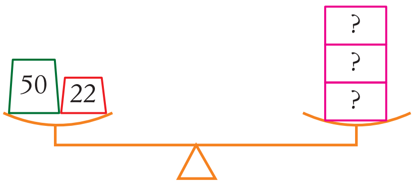

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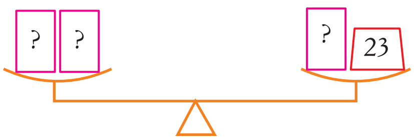

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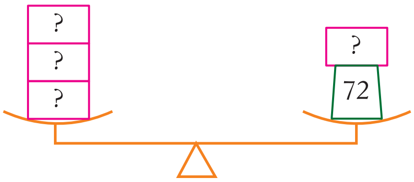

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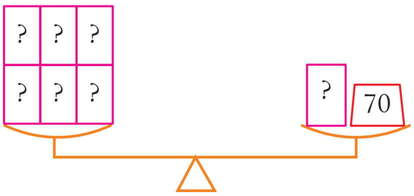

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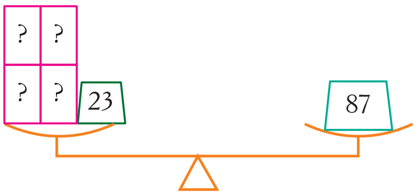

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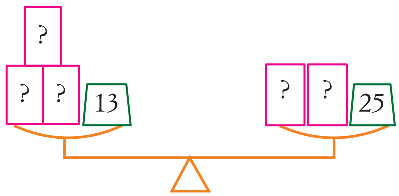
Worked solutions
Exercise 6.4
MAT07NAWS00004

6 **a**



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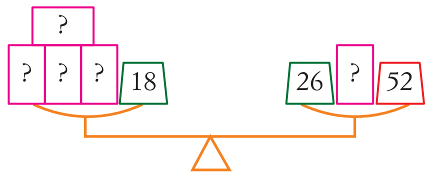
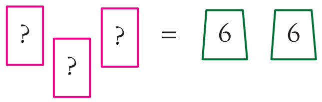


For questions **7** to **9**, the level balance has been replaced by an equals sign. Find the masses of the parcels marked '?'.

7 **a**



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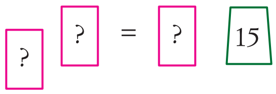
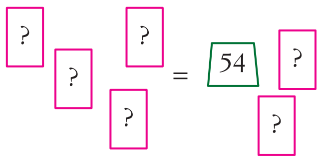


8 **a**



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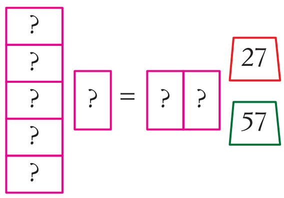
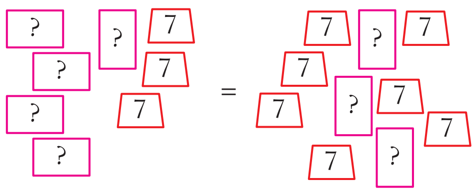


9 **a**



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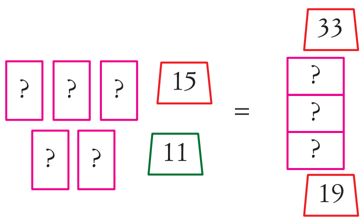

10 Find the value of the variable that makes the following equations true.

See Example 23

a $d + 1 = 7$ **b** $3 + m = 14$ **c** $a - 3 = 10$ **d** $6 - c = 2$

e $3e = 15$ **f** $\frac{12}{f} = 4$ **g** $s + 2s = 9$ **h** $15 - j = 10$

i $8q = 16$ **j** $g - 6 = 8$ **k** $5 + i = 12$ **l** $3k = 15$

m $5x - 2x = 18$ **n** $a + 4a = 25$ **o** $\frac{p}{3} = 9$ **p** $3z - z = 8$

Fluency

11 Is the number in brackets a solution to the equation (yes or no)?

See Example 22

a $a + 4 = 7$ (3) **b** $m - 6 = 3$ (9) **c** $15 - n = 7$ (5)
d $f + 4 = 11$ (6) **e** $10 - g = 6$ (4) **f** $2h + 3 = 17$ (6)
g $3d - 4 = 11$ (5) **h** $2b + 3 = 17$ (7) **i** $8 - 4z = 16$ (2)
j $11 + c = 7$ (4)

12 Write the following equations as flow charts, rearranging if necessary.

See Example 24

a $3h + 4 = 19$ **b** $5y - 7 = 13$ **c** $27 = 7u + 6$

d $5 = 6j - 13$ **e** $\frac{k + 6}{5} = 4$ **f** $\frac{p - 8}{3} = 11$

13 Solve each of the equations in question **12** by backtracking the flow charts you formed.

See Example 25

14 Put in the inverse operations for each of the following flow charts.

a $h \xrightarrow{\times 3} \square \xrightarrow{-8} 1$

b $S \xrightarrow{\times 5} \square \xrightarrow{+12} 22$

c $b \xrightarrow{\div 2} \square \xrightarrow{-9} 5$

d $k \xrightarrow{\div 4} \square \xrightarrow{+11} 15$

e $M \xrightarrow{\times 4} \square \xrightarrow{-16} \square \xrightarrow{\div 5} 4$

f $f \xrightarrow{\div 3} \square \xrightarrow{+7} \square \xrightarrow{\times 2} 22$

15 Solve each of the flow charts in question **14** by backtracking.

See Example 26

Worked solutions
Exercise 6.4
MAT07NAWS00004

16 Use flow charts and backtracking to solve the following equations.

a $2d + 6 = 22$
b $3k - 17 = 13$
c $\frac{r}{3} + 5 = 12$
d $2(m + 3) = 16$
e $3(x - 8) = 27$
f $\frac{g + 4}{7} = 3$
g $7n + 11 = 60$
h $48 = 6e - 12$
i $\frac{4f - 9}{3} = 13$

17 Give the inverses of the following operations.

a *Walk* in the door.
b *Pick up* a book.
c *Add* 3.
d *Subtract* 2.
e Look *up*.
f *Multiply* by 7.
g *Divide* by 3.
h Fall *down*.
i *Take off* from the airstrip.
j *Put on* a hat.

18 Copy and complete the following.

a $7 - \square = 0$
b $\square - 4 = 0$
c $8 \div \square = 1$
d $\frac{1}{4} \times \square = 1$
e $12 - \square = 0$
f $11 \times \square = 1$
g $2 \div \square = 1$
h $a - \square = 0$
i $\square - d = 0$
j $m \times \square = 1$

19 What must be done to each expression to get *b* by itself?

a $b - 6$
b $8b$
c $b + 5$
d $b \div 3$
e $b \times 10$
f $7 + b$
g $\frac{b}{4}$
h $b - 12$
i $b + a$
j $b \times c$

20 Copy and complete the following.

a $d + 3 - \_\_\_\_ = d$
b $d \times 4 \div \_\_\_\_ = d$
c $d - 4 \_\_\_\_ = d$
d $3d \_\_\_\_ = d$
e $4 + d \_\_\_\_ = d$
f $d \div 7 \_\_\_\_ = d$
g $\frac{d}{13} \_\_\_\_ = d$
h $d \times \frac{1}{2} \_\_\_\_ = d$
i $d + a \_\_\_\_ = d$
j $\frac{d}{m} \_\_\_\_ = d$

21 Solve these equations by using the instructions given.

a $x + 14 = 20$ (Subtract 14 from both sides.)
b $y - 7 = 4$ (Add 7 to both sides.)
c $3w = 12$ (Divide both sides by 3.)
d $\frac{m}{12} = 3$ (Multiply both sides by 12.)
e $9 + d = 17$ (Subtract 9 from both sides.)
f $q \div 12 = 6$ (Multiply both sides by 12.)

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22 Copy and complete the following.

a
$$a + 5 = 14$$
$$a + 5 - \square = 14 - \square$$
$$a = \square$$

b
$$6b = 18$$
$$\frac{6b}{\square} = \frac{18}{\square}$$
$$b = \square$$

c
$$d - 12 = 21$$
$$d - 12 + \square = 21 + \square$$
$$d = \square$$

d
$$\frac{f}{7} = 13$$
$$\frac{f}{7} \times \square = 13 \times \square$$
$$f = \square$$

23 Solve these equations by using inverse operations. *See* Examples 27, 28

a $5p = 35$ **b** $x - 10 = 8$ **c** $\frac{z}{8} = 3$ **d** $s + 4 = 5$

e $d - 5 = 6$ **f** $8f = 32$ **g** $y - 13 = 7$ **h** $b \div 4 = 12$

i $p \times 7 = 49$ **j** $36 \times g = 72$ **k** $a - 14 = 20$ **l** $\frac{h}{12} = 7$

m $9i = 81$ **n** $9 + f = 17$ **o** $\frac{u}{8} = 8$

For questions **24** and **25** below, $\square$ indicates that one or more numbers and/or operations need to be completed.

24 Use inverse operations to complete the following steps for solving each equation.

a
$$3h + 4 = 22$$
$$3h + 4 \square = 22 \square$$
$$\frac{3h}{\square} = \frac{\square}{\square}$$
$$h = \square$$

b
$$5m + 3 = 28$$
$$5m + 3 \square = 28 \square$$
$$\frac{5m}{\square} = \frac{\square}{\square}$$
$$m = \square$$

c
$$2p - 4 = 12$$
$$2p - 4 \square = 12 \square$$
$$\frac{2p}{\square} = \frac{\square}{\square}$$

25 Use inverse operations to complete the following steps for solving each equation.

a
$$\frac{c+4}{5} = 3$$
$$\frac{c+4}{5} \square = 3 \square$$
$$c + 4 = \square$$
$$c + 4 \square = \square$$
$$c = \square$$

b
$$\frac{k+2}{7} = 5$$
$$\frac{k+2}{7} \square = 5$$
$$k + 2 = \square$$
$$k + 2 \square = \square$$
$$k = \square$$

c
$$\frac{r-5}{3} = 8$$
$$\frac{r-5}{3} \square = 8 \square$$
$$r - 5 = \square$$
$$r - 5 \square = \square$$
$$r = \square$$

26 Solve these equations. *See* Example 29

a $7a + 9 = 30$ **b** $3s + 6 = 18$ **c** $3y - 1 = 8$

d $2x + 3 = 15$ **e** $3e - 12 = 6$ **f** $10p - 2 = 98$

g $4 = 3m - 2$ **h** $5w - 8 = 2$ **i** $5y + 14 = 89$

j $2d - 7 = 1$ **k** $44 = 9 + 7x$ **l** $10 + 4m = 34$

m $5n - 14 = 16$ **n** $5b + 7 = 42$ **o** $3c - 8 = 16$

See Example 30

27 Solve these equations by first using the distributive law.

a $2(a + 1) = 6$ **b** $4(b + 3) = 12$ **c** $3(c - 4) = 27$
d $3(f - 10) = 18$ **e** $4(2g + 3) = 36$ **f** $42 = 6(3h - 2)$
g $5(3k + 2) = 25$ **h** $8(2j - 1) = 64$ **i** $2(m + 3) - 7 = 19$

Problem solving

28 Find the value of the variable in each of the following.

a x is the number of eggs in a dozen.
b y is the number of wheels on a car.
c p is the number of runs in a century in cricket.
d m is the number of days in a week.
e r is the number of letters in the English alphabet.
f q is the number of points for a goal in netball.
g t is the number of cards in a pack without jokers.

See Example 30

29 For each of the following, write an equation that shows the information.

a When 6 is added to a number, the answer is 11.
b When a number is reduced by 7, the result is 8.
c Three times a number is 12.
d Half a number is 9.
e When a number is doubled, the result is 16.
f Multiply a number by 3 and add 1. The result is 10.
g Multiply a number by 4 and subtract 5. The result is 19.

30 Work out the number in each case in question **29**.

Reasoning

31 Write each of the following as an equation and solve using inverse operations.

a The sum of a number and eight is 6.
b The product of a number and 7 is 42.
c The difference between a number and ten is 3.
d The quotient of a number and 5 is 35.
e The sum of double a number and 7 is 29.

Worked solutions
Exercise 6.4
MAT07NAWS00004

32 Write each of the following as an equation and solve using inverse operations.

a Three times the sum of a number and 5 is 36.
b The difference between seven times a number and 5 is 51.
c The quotient of the sum of eight and a number and nine is 4.
d The product of 4 and the sum of twice a number and 3 is 96.
e The sum of a third of a number and 20 is 2.

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33 The following are the solutions of equations by a student. In each case there is an error. State the line on which the first error occurred and the correct answer for each one.

a $3x - 6 = 18$

$3x = 18 - 6$ A

$3x = 12$ B

$x = 4$ C

b $4x + 8 = 32$

$x + 2 = 8$ A

$x = 8 + 2$ B

$x = 10$ C

c $2 - 7x = 30$

${}^{-}7x = 30 - 2$ A

${}^{-}7x = 28$ B

$x = 4$ C

d $2x + 3 = 31$

$2x = 31 + 3$ A

$2x = 34$ B

$x = 17$ C

e $3(x + 2) = 24$

$3x + 6 = 24$ A

$3x = 24 - 6$ B

$x = 2$ C

f $4(x + 8) = 16$

$4x + 8 = 16$ A

$4x = 8$ B

$x = 2$ C

Chapter 6 summary

Quiz
Expressions and equations
MAT07NAQZ00001

Weblink
Algebra Help

- A number pattern consists of **terms**. The **term number** shows which term it is; the second, third and fifth terms of 4, 7, 10, 13, 16, 19, 25, … are 7, 10 and 16 respectively.
- The **rule** for a number pattern is written with the term number as the **input** and the term as the **output** number. A number pattern can be written as a table with the input numbers in the first row and the output numbers in the second row.
- A **variable** (**pronumeral** or **unknown**) is a letter or symbol that stands for a number. The rule for a number pattern can be written with a variable for the term number.
- A **linear** number pattern goes up or down by the same amount from one term to the next.
- Addition and multiplication are both **commutative**: when two numbers are added or subtracted, the order makes no difference, so $a + b = b + a$ and $a \times b = b \times a$ for any numbers.
- Addition and multiplication are both **associative**. When the operation is performed on three numbers, it does not matter which pair is done first, so $a + b + c = (a + b) + c = a + (b + c)$ and $a \times b \times c = (a \times b) \times c = a \times (b \times c)$ for any three numbers.
- The commutative and associative properties are called **laws** because they work for all numbers.
- The **distributive law** states that the product of the sum (or difference) of two numbers with another number is the same as the sum (or difference) of the products of that number with each one separately. This can be written in symbols as:

$$a \times (b + c) = a \times b + a \times c \quad \text{and} \quad a \times (b - c) = a \times b - a \times c.$$

- The removal of brackets using the distributive law is called **expanding brackets**.
- An **arithmetic expression** has numbers connected by arithmetic operations such as − and ×.
- An **algebraic expression** includes variables. Replacement of the variables by values is called **substitution**.
- A **constant** is a number in an algebraic expression. The numbers that are multiplied by the variables are called **coefficients**, the parts *separated* from the rest by + or − are called **terms**, and a number on its own is called a **constant term**.
- If an algebraic expression contains terms with exactly the same variables, we call these **like terms**. Like terms can be added or subtracted to **simplify** the expression.
- An **equation** is a mathematical sentence. It has an equals sign with expressions on both sides. The expressions can be arithmetic or algebraic.
- Calculation of the answer to an expression is called **evaluation** of the expression.
- A **solution** of an algebraic equation is a value of the variable that makes the equation true. A solution makes the left-hand side (**LHS**) equal to the right-hand side (**RHS**).
- The process of finding the solution to an equation is called **solving** an equation.
- The '**guess and check**' (**trial and error**) method of solving equations involves the evaluation of both sides of the equation with different values of the variable until a solution is found.
- An equation with only a number on the right-hand side can be shown as a **flow chart**. The operations are shown in the correct order on arrows between boxes that contain the values at each step. The variable is shown in the first box and the right-hand side value is in the last box.
- The **backtracking** method solves an equation by working backwards on a flow chart.
- **Inverse operations** are opposites. The inverse of × is ÷ and the inverse of − is +. They can be used to solve an equation by isolating the variable on the left-hand side.

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Understanding

1 Write a rule in words for each of the following number patterns and work out the indicated term. See Example 1

a 3, 8, 13, 18, 23, …, 12th term

b 7, 11, 15, 19, …, 20th term

2 Write a rule in words for each of the following number patterns. See Example 2

Worksheet
Expressions and equations
MAT07NAWK00004

a

Input	1	2	3	4	5
Output	5	9	13	17	21

b

Input	3	4	5	6	7
Output	7	10	13	16	19

3 For the number pattern 3, 9, 15, 21, 27, … See Example 3

a write a rule using a box (□) for the term number

b work out the 11th term.

4 What is the coefficient of $^{-}3x^4z$? See Example 14

5 How many terms are in the expression $^{-}3x - 5y + 8 + z$? See Example 15

6 For the number pattern 4, 10, 16, 22, 28, … use the variable n to write the rule and find the 15th term. See Examples 4, 5

7 Work out these problems in your head. See Examples 8, 9

a $20 \times 123 \times 5$

b $16 + 28 + 12 + 34$

8 How many terms are there in the expression $3(2x + 5y) - 3z + 2$? See Example 15

9 What are the like terms in the set $3p$, $3q$, $5pq$, $^{-}2p$, $6q$?

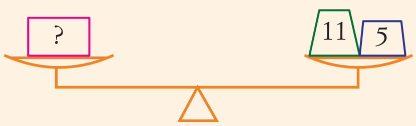
10 Simplify each of the following. See Examples 17, 18

a $7e + 5e$

b $3k - 2m + 6m + 4k - 1$

11 For this question, the masses shown on the balances are in kilograms. For each part, all masses marked with ? are the same. Find the masses of the parcels marked. See Example 21

a



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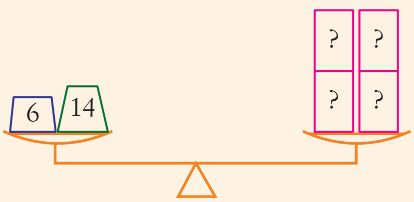
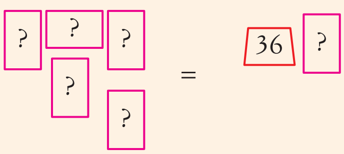
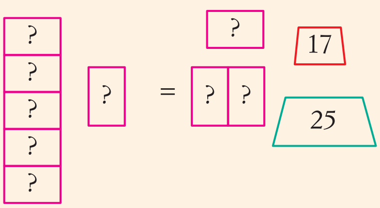


c



d



12 Is the number in brackets a solution to the equation (yes or no)? See Example 22

a $k + 8 = 15$ (7)

b $6h = 18$ (4)

See Example 23

13 Find the values of the variables that make each of the following equations true.

a $t + 14 = 27$ **b** $4u = 36$

See Example 24

14 Write the following equations as flow charts.

a $4p + 8 = 27$ **b** $\frac{k - 11}{2} = 7$

See Example 25

15 Insert inverse operations in each of the following flow charts.

a $y \xrightarrow{\times 4} \square \xrightarrow{-12} 20$ **b** $f \xrightarrow{\div 6} \square \xrightarrow{+11} 17$

See Example 26

16 Use flow charts and backtracking to solve the following equations.

a $2h + 5 = 17$ **b** $4m - 14 = 18$ **c** $\frac{k}{5} + 3 = 7$ **d** $\frac{3r - 5}{2} = 8$

Fluency

17 For the number pattern 44, 41, 38, 35, 33, … use the variable n to write the rule and find the 20th term.

18 Write rules in symbols for the following tables of inputs and outputs.

a

f	1	2	3	4	5	6
h	2	9	16	23	30	37

b

m	1	2	3	4	5	6
p	99	95	91	87	83	79

See Examples 12, 13

19 Expand the brackets and simplify each of the following.

a $5(d + 8)$ **b** $6(2h - 3p)$

20 Simplify each of the following.

a $7e^2 - 5e + e^2 - 2e$ **b** $6x + 5y - 3 + y + 8 - 7y - 9z$

21 Simplify each of the following.

a $3x \times {}^{-}5y$ **b** $24pqr^3 \div 6pr$ **c** $\frac{18vj^2}{24jv^2}$

See Examples 27–29

22 Solve these equations using inverse operations.

a $9g + 5 = 41$ **b** $\frac{8p}{3} - 5 = 11$

See Example 30

23 Solve these equations by first using the distributive law.

a $3(m + 2) = 15$ **b** $2(3w - 11) = 20$

Problem solving

24 Peter says "5, 8, 11, 14, 17". What would be the 10th number he says?

25 A life raft has 75 litres of water on board. Each person on the life raft drinks 1.8 litres of water each day. There are 4 people on the life raft. How much water is left after 4 days? How long will the water last?

26 Write down and explain a short cut for working out 171×99.

27 David was paid \$34 for every ten books he sold at a bookshop. On Tuesday he sold 180, on Wednesday 60 and on Thursday 120. Explain how to use a mathematical law to work out how much he was paid.

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28 Write expressions for:

a the difference between 9 and g

b 40 less the product of f and 7

c the number of men and women at a wedding.

29 Write each of the following as an equation and solve to find the number.

a The sum of a number and 7 is 20.

b The difference between triple a number and 8 is 7.

30 Write a formula to find the total number of seconds, t, in m minutes and s seconds. Use your formula to find the total number of seconds in 3 minutes 21 seconds.

Reasoning

31 The daily hiring cost of a pneumatic drill is a linear rule of the number of days. It costs \$59 to hire for 2 days and \$95 to hire for 5 days. How much would it cost for 11 days?

32 Expand and simplify $-(6x - 5)$.

33 Work out a rule for multiplying a two-digit number by 201 and explain the rule.

34 Write each of the following as an equation and solve using inverse operations.

a The quotient of a number and 5 is $^{-}3$.

b The product of a number and 3 is 5 less than 29.